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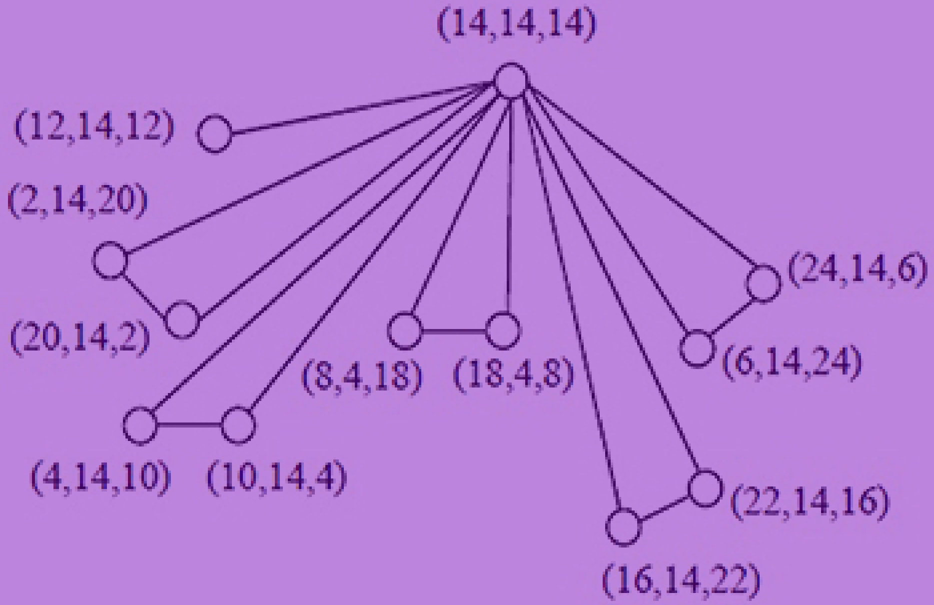

NEUTROSOPHIC TRIPLET GROUPS AND THEIR APPLICATIONS TO MATHEMATICAL MODELLING

Neutrosophic Triplet Groups and their Applications to Mathematical Modelling

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PREFACE

The innovative notion of neutrosophic triplet groups, introduced by Smarandache and Ali in 2014-2016, happens to yield the anti-element and neutral element once the element is given. It is established that the neutrosophic triplet group collection forms the classical group under product for Z_n, for some specific n. However the collection is not even closed under sum. These neutrosophic triplet groups are built using only modulo integers or Cayley tables.

Several interesting properties related with them are defined. It is pertinent to record that in Z_n, when n is a prime number, we cannot get a neutral element which can contribute to nontrivial neutrosophic triplet groups. Further, all neutral elements in Z_n are only nontrivial idempotents.

Using neutrosophic triplet groups authors have defined the notion of neutrosophic triplet group matrices. Further as the notion of operation addition or max or min cannot be defined on

these triplet groups; authors have overcome this problem by defining the new notion of conditionally neutral minimum (c.n. min) and conditionally neutral maximum (c.n. max) for min and max operations respectively. However the operation of addition can never be compatible.

We define these new operations mainly to construct mathematical models akin to Fuzzy Cognitive Maps (FCMs) model, Neutrosophic Cognitive Maps (NCMs) model and Fuzzy Relational Maps (FRMs) model. These new models are defined in chapter four of this book. These new models can find applications in discrete Artificial Neural Networks, soft computing, and social network analysis whenever the concept of indeterminate is involved.

The Neutrosophic Duplets were introduced by Smarandache in 2016.

Further authors have defined algebraic codes in a special way as automatically these codes built using Z_n lead to mutually orthogonal codes or dual codes. Study in this direction is open.

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Chapter One

INTRODUCTION TO NEUTROSOPHIC TRIPLET GROUPS

The innovative study of neutrosophic triplet groups was first started by Florentin and Ali in [7]. These neutrosophic triplet groups satisfy certain algebraic properties. However it is recorded in [7] that these triplets collection do not enjoy the classical group structure.

In fact they have proved the collection of all neutrosophic triplet groups form a semigroup under product. We recall all definitions from [7].

Definition 1.1. *Let N be a set together with a binary operation *. Then N is called a neutrosophic triplet set if for any a $\in$ N there exists a neutral 'a' called neut(a) different from the classical algebraic unitary element and an opposite of 'a' called anti (a) with neut (a) and anti (a) belonging to N, such that*

$$a * neut(a) = neut(a) * a = a$$

and $a * anti(a) = anti(a) * a = neut(a)$.

The elements a, neut (a) and anti(a) are collectively called as neutrosophic triplet groups and we denote it by (a, neut(a), anti(a)). By neut (a), we mean neutral of a and apparently a is just the first coordinate of a neutrosophic triplet and not a neutrosophic triplet.

For the same element a, 1 in N there may be more neutrals to it neut (a) and more opposite of it anti(a).

Definition 1.2 . *The element b in (N, *) is the second component denoted by neut (.) of a neutrosophic triplet if there exists other elements a and c in N such that* $a * b = b * a = a$ *and* $a * c = c * a = b$.

The resultant neutrosophic triplet is (a, b, c).

***Definition 1.3**. The element c in (N, *) is the third component denoted by anti(.) of a neutrosophic triplet, if there exists other elements a and b in N such that* $a * b = b * a = a$ *and* $a * c = c * a = b$. *The formed neutrosophic triplet is (a, b, c).*

We will illustrate this by examples.

Example 1.1. Let $\{Z_{14},\times\}$ be the semigroup under multiplication modulo 14. The only idempotents in Z_{14} are 7 and 8. We further see none of the elements 1 3, 5, 9, 11 and 13 in Z_{14} contribute to neutrosophic triplets as they are units under product in Z_{14}.

Only elements 2, 4, 6, 8, 10 and 12 are the probable ones in Z_{14} which can contribute to the neutrosophic triplet groups.

We see $2 \in Z_{14}$ is such that $2 \times 8 = 2 \pmod{14}$ and 8 is the neut(2). Further $2 \times 4 = 8 \pmod{14}$ so 4 is the anti (2). Thus (2, 8, 4) forms the neutrosophic triplet. neut (2) = 8 and anti (2) = 4.

Similarly $4 \times 8 \equiv 4 \pmod{14}$ and $4 \times 2 = 8 \pmod{14}$ so (4, 8, 2) is also a neutrosophic triplet neut (4) = 8 and anti (4) = 2.

Consider $6 \in Z_{14}$, $6 \times 8 = 6 \pmod{14}$ so neut(6) = 8 and $6 \times 6 = 8$ so anti (6) = 6. Thus (6, 8, 6) is a neutrosophic triplet.

Now for $10 \in Z_{14}$ $10 \times 8 = 10 \pmod{14}$ so neut(10) = 8 and anti(10) = 8 so (10, 8, 12) is a neutrosophic triplet.

For $12 \times 8 = 12$ so neut(12) = 8 and anti(12) = 10 hence (12, 8, 10) is also a neutrosophic triplet.

Clearly $7 \times 7 = 7 \pmod{14}$ but 7 is not a neutral element further (0, 0, 0) and (7, 7, 7) are trivial neutrosophic triplets. We see $K = \{2, 4, 6, 8, 10, 12\} \subseteq Z_{14}$ is such that they form a group under product modulo 14, with 8 as the identity given by the following table.

×	2	4	6	8	10	12
2	4	8	12	2	6	10
4	8	2	10	4	12	6
6	12	10	8	6	4	2
8	2	4	6	8	10	12
10	6	12	4	10	2	8
12	10	6	2	12	8	4

However we give yet another example.

Example 1.2. Let S = $\{Z_{15}, \times\}$ be the semigroup under product modulo 15. 6 and 10 are the only non-trivial idempotents of Z_{15}, $6 \times 6 = 6 \pmod{15}$ and $10 \times 10 = 10 \pmod{15}$.

Let $3 \in Z_{15}$, $3 \times 6 = 3 \pmod{15}$, neut (3) = 6, and anti (3) = 12 as $3 \times 12 \equiv 6 \pmod{15}$.

Thus (3, 6, 12) is a neutrosophic triplet group.

For $12 \in Z_{15}$, $12 \times 6 = 12 \pmod{15}$, neut (12) = 6 and $12 \times 3 \equiv 6 \pmod{15}$ hence anti (12) = 3.

Hence (12, 6, 3) is a neutrosophic triplet (6, 6, 6) is also a neutrosophic triplet.

For $5 \in Z_{15}$ we have $5 \times 10 \equiv 5 \pmod{15}$ and neut (5) = 10. Now $5 \times 14 = 10 \pmod{15}$ so anti (5) = 14.

However (14, 10, 5) is not a neutrosophic triplet as $14 \times 10 \equiv 5 \pmod{15}$ and 14 is a unit of Z_{15}. (9, 6, 9) is a neutrosophic triplet.

For $9 \in Z_{15}$, $9 \times 6 = 9 \pmod{15}$ and $9 \times 14 = 6 \pmod{15}$.

So (9, 6, 14) is not a neutrosophic triplet and (14, 6, 9) is also not a neutrosophic triplet.

We see {3, 6, 12, 9} forms a semigroup with 6 as the neutral element.

We give the table of {3, 6, 12, 9} under $\times$.

$\times$	3	6	12	9
3	9	3	6	12
6	3	6	12	9
12	6	12	9	3
9	12	9	3	6

Consider the table for {5, 10, 14}

$\times$	5	10	14
5	10	5	10
10	5	10	5
14	10	5	1

We see {5, 10, 14} is not even closed under product so will not form a semigroup so $10 \in Z_{15}$ is not neutral element as $5 \times 10 = 5$ but $5 \times 14 = 10$ so this sort of neutrosophic triplets behave very differently and we do not in general define them as neutrosophic triplet as $14 \times 14 = 1 \pmod{15}$ is a unit in Z_{15}.

Example 1.3. Let $\{Z_{18}, \times\}$ be the semigroup under product modulo 18. 9 and 10 are the only idempotents of Z_{18}.

$2 \times 10 = 2 \pmod{18}$,

$2 \times 14 = 10 \pmod{18}$ so (2, 10, 14) and (14, 10, 2) are neutrosophic triplets of Z_{18}. 3 does not contribute to neutrosophic triplets. (4, 10, 16) and (16, 10, 4) are

neutrosophic triplets and (8, 10, 8) is again a neutrosophic triplet.

It is unusual, 3, 6, 12 and 15 do not contribute to any neutrosophic triplet (9, 9, 9) is the trivial neutrosophic triplet.

So H = {(4, 10, 16), (16, 10, 4), (10, 10, 10), (8, 10, 8), (2, 10, 14) (14, 10, 2)} forms the collection of non trivial neutrosophic triplets.

Clearly (10, 10, 10) acts as the identity element of H. The table for H is as follows.

×	(4, 10, 16)	(16, 10, 4)	(10, 10, 10)
(4,10,16)	(16,10,4)	(10,10,10)	(4,10,16)
(16,10,4)	(10,10,10)	(4,10,16)	(16,10,4)
(10,10,10)	(4,10,16)	(16,10,4)	(10,10,10)
(8,10,8)	(14,10,2)	(2,10,14)	(8,10,8)
(2,10,14)	(8,10,8)	(14,10,2)	(2,10,14)
(14,10,2)	(2,10,14)	(8,10,8)	(14,10,2)

(8, 10, 8)	(2,10,14)	(14,10,2)
(14,10,2)	(8,10,8)	(2,10,14)
(2,10,14)	(14,10,2)	(8,10,8)
(8,10,8)	(2,10,14)	(14,10,2)
(10,10,10)	(16,10,4)	(4,10,16)
(16,10,4)	(4,10,16)	(10,10,10)
(4,10,16)	(10,10,10)	(16,10,4)

H is a group of order six.

Clearly (2, 10, 14) generates H as

(2, 10, 14) × (2, 10, 14) = (4, 10, 16),

(2, 10, 14) (4, 10, 16) = (8, 10, 8),

(2, 10, 14) (8, 10, 8) = (16, 10, 4),

(2, 10, 14) × (16, 10, 4) = (14, 10, 2),

and (2, 10, 14) × (14, 10, 2) = (10, 10, 10).

Thus $(2, 10, 14)^6 = (10, 10, 10)$.

Hence H is a cyclic group of order six.

Example 1.4. Let S = $\{Z_{50}, \times\}$ be the semigroup under product modulo 50. 25 and 26 are the only idempotents of Z_{50}.

The neutrosophic triplet groups associated with the neutral element 26 are

H = {(2, 26, 38), (38, 26, 2), (4, 26, 44),

(44, 26, 4), (8, 26, 22), (22, 26, 8)

(6, 26, 46) (46, 26, 6) (12, 26, 48)

(48, 26, 12), (14, 26, 34), (34, 26, 14)

(16, 26, 36), (36, 26, 16), (18, 26, 32),

(32, 26, 18), (24, 26, 24), (28, 26, 42)

(42, 26, 28), (26, 26, 26)}.

We see {(2, 26, 38)} generates the group H and H is a cyclic group of order 20. We see 26 acts as the identity for K = {2, 4, 8, 12, 16, 32, 14, 28, 6, 24, 48, 46, 42, 34, 18, 36, 22, 44, 38, 26} and $2^{20} = 26$ that is 2 generates this cyclic group.

We see though 5, 10, 15, 20, 30, 40, 35 and 45 are not units still they do not contribute to neutrosophic triplet groups.

We see $2 \times 5^2 = 50$.

Next we consider another example.

Example 1.5. Let S = $\{Z_{20}, \times\}$ be the semigroup under product modulo 20. Here the neutrosophic triplets are formed in a very unique way.

We see the only idempotents in Z_{20} are 5 and 16 as

$5 \times 5 = 5 \pmod{20}$ and $16 \times 16 = 16 \pmod{20}$.

We see 2 is not neutral as $2 \times 5 = 10 \pmod{20}$ and $2 \times 16 = 12 \pmod{20}$.
Consider $4 \in Z_{20}$;

$4 \times 5 = 0 \pmod{20}$ so 5 is not a neutral of 4 and

$4 \times 16 = 4 \pmod{20}$ so 16 is the neutral of 4.

Now $4 \times 4 = 16$ so anti (4) = 4.

Thus (4, 16, 4) is a neutrosophic triplet associated with Z_{20}.

Consider $6 \in Z_{20}$, $6 \times 5 = 10 \pmod{20}$ so 5 is not a neutral of 6.

Also $6 \times 16 = 16 \pmod{20}$ so 16 is also not a neutral of 6. Thus 6 does not form a neutrosophic triplet group.

Consider $8 \in Z_{20}$, $8 \times 5 = 0 \pmod{20}$ so 5 is not a neutral of 8. Now $8 \times 16 \equiv 8 \pmod{20}$ so 16 is a neutral of 8 and anti 8 is 12.

Consider $10 \in Z_{20}$, $10 \times 5 = 10 \pmod{5}$ and there is no anti for 10.

So 10 cannot contribute to a neutrosophic triplet with 5.

$10 \times 16 \equiv 0 \pmod{20}$ so 10 cannot contribute for a neutrosophic triplet.

Consider $12 \in Z_{20}$ $12 \times 10 \equiv 0 \pmod{20}$ so 10 is not a neut (12).

$12 \times 16 = 12$ so neut(12) = 16 and anti(12) is 8.

Thus (8, 16, 12) and (12, 16, 8) are neutrosophic triplets.

Now $15 \in Z_{20}$; $15 \times 5 = 15 \pmod{20}$ so neut (15) = 5 and $15 \times 5 = 5 \pmod{20}$ so anti (15) = 15. Hence (15, 5, 15) is a neutrosophic triplet.

Finally $18 \in Z_{20}$, $18 \times 5 = 10 \pmod{20}$ so 5 is not a neutral of 18.

$18 \times 16 \neq 19 \pmod{20}$ so 16 is not the neuter of 18 anti 18 cannot be 2 or 12 so (18, 16, 2) and (18, 16, 12) are not neutrosophic triplets.

Thus we get the following collection of neutrosophic triplets {(4, 16, 4), (8, 16, 12), (8, 16, 2), (12, 16, 8), (16, 16, 16), (15, 5, 15), (5, 5, 5)}.

We see (4, 16, 4) $\times$ (4, 16, 4) = (16, 16, 16).

(4, 16, 4) $\times$ (8, 16, 12) = (8, 16, 8).

(4, 16, 4) $\times$ (12, 16, 8) = (8, 16, 8).

However (8, 16, 8) is not a neutrosophic triplet as $8 \times 16 = 8 \pmod{16}$ but $8 \times 8 \neq 16$ is not possible hence the claim.

(4, 16, 4) $\times$ (15, 5, 15) = (0, 0, 0) the trivial neutrosophic triplet. (8, 16, 12) $\times$ (15, 5, 15) = (0, 0, 0) and (12, 16, 12) $\times$ (15, 5, 15) = (0, 0, 0).

So the collection is not even closed under product when $\{Z_{20}, \times\}$ is used as a semigroup under product modulo 20.

Example 1.6. Let $\{Z_{45}, \times\}$ be the semigroup under product modulo 45. Only 10 and 36 are the idempotents of Z_{45}. (9, 36, 9) and (36, 36, 36) are neutrosophic triplets of Z_{45}.

$15 \times 10 = 15 \pmod{45}$.

$15 \times x = 10$ we cannot get any x such that $15x \equiv 10 \pmod{45}$ that is 15 has no anti (45), only neut (45) = 10.

For $18 \times Z_{45}$, $18 \times 36 = 18 \pmod{45}$.

$18 \times 26 = 36 \pmod{45}$.

Hence (18, 36, 27) and (27, 36, 18) are neutrosophic triplets associated with 36. (5, 10, 20) and (20, 10, 5) are triplets associated with Z_{45}. $30 \times 10 \equiv 30 \pmod{45}$, however finding anti 30 is a difficult task. (10, 10, 10) is also a neutrosophic triplet.

Interested reader can find all nontrivial neutrosophic triplets associated with Z_{45}.

Example 1.7. Let $S = \{Z_{30}, \times\}$ be the semigroup under $\times$ modulo 30.

We have mainly considered this example as there are six idempotents in Z_{30}, 6, 10, 15, 16, 21 and 25.

We see (2, 16, 8) and (8, 16, 2) are neutrosophic triplets associated with $16 \in Z_{30}$. (3, 21, 27) and (27, 21, 3) are neutrosophic triplets associated with $21 \in Z_{30}$. (4, 16, 4) is a neutrosophic triplet. (5, 25, 5) is a neutrosophic triplet. (9, 21, 9) is again a neutrosophic triplet. (12, 6, 18) and (18, 6, 12) are neutrosophic triplets. (14, 16, 14) is a neutrosophic triplet. (20, 10, 20) is neutrosophic triplet.

We see (22, 16, 28) and (28, 16, 22) are neutrosophic triplets associated with $16 \in Z_{30}$ and (24, 6, 24) is a neutrosophic triplet. (16, 16, 16), (6, 6, 6), (21, 21, 21), (25, 25, 25) and (10, 10, 10) are neutrosophic triplets. (26, 16, 26) is again a neutrosophic triplet.

Thus we see Z_{30} is very unique for it has six idempotents and 20 neutrosophic triplets. It is important to note 15 does not yield any neutrosophic triplet so (15, 15, 15) is only a trivial neutrosophic triplet.

We are yet to study the structure of the collection of all neutrosophic triplets of Z_{30}.

We see in the first place it is not closed under product.

For $(5, 25, 5) \times (18, 6, 12) = (0, 0, 0)$,

$(2,16,8) \times (2, 16, 8) = (4, 16, 4)$,

$(2, 16, 8) \times (3, 21, 27) = (6, 6, 6)$,

$(3, 21, 27) \times (6, 6, 6) = (18, 6, 12)$,

$(18, 6, 12) \times (3\ 21, 27) = (24, 6, 24)$,

$(24, 6, 24) \times (3, 21, 27) = (12, 6, 18)$,

$(12, 6, 18) \times (3, 21, 27) = (6, 6, 6)$ and

$(5, 25, 5) \times (6 \times 6 \times 6) = (0, 0, 0)$;

thus (5, 5, 5) and (5, 25, 5) annuls all neutrosophic triplets except (15, 15, 15) for $(15, 15, 15) \times (5, 5, 5) = (15, 15, 15)$, $(15, 15, 15) \times (5, 25, 5) = (15, 15, 15)$ infact (15, 15, 15) acts as the identity.

Further $(25, 25, 25) \times (15, 15, 15) = (15, 15, 15)$.

$(25, 25, 25) \times (5, 25, 5) = (5, 25, 5)$.

$(15, 15, 15) \times (15, 15, 15) = (15, 15, 15)$.

So in $Z_{n,}$ where $n = p_1\ p_2 \ldots p_t$ where p_i's are distinct primes then the associated neutrosophic triplets behave in a unique way.

Example 1.8. Let S = {Z_{35}, ×} be the semigroup under product modulo 35.

The idempotents in Z_{35} are 15 and 21. (5, 15, 10) and

(10, 15, 5) are neutrosophic triplets associated with 15.

(7, 21, 28) and (28, 21, 7) are neutrosophic triplets of the neutral element 21.

(14, 21, 14) is a neutrosophic triplet. (20, 15, 20) is a neutrosophic triplet.

(25, 15, 30) and (30, 15, 25) are neutrosophic triplets.

(15, 15, 15) and (21, 21, 21) are again neutrosophic triplet groups.

Consider the two tables of neutrosophic triplets under product.

×	(20,15,20)	(30,15,25)	(25,15,30)
(20,15,20)	(15,15,15)	(5,15,10)	(10,15,5)
(30,15,25)	(5,15,10)	(25,15,30)	(15,15,15)
(25,15,30)	(10,15,5)	(15,15,15)	(30,15,25)
(15,15,15)	(20,15,20)	(30,15,25)	(25,15,30)
(5,10,10)	(30,15,25)	(10,15,50)	(20,15,20)
(10,15,5)	(25,15,30)	(20,15,20)	(5,15,10)

(15,15,15)	(5,15,10)	(10,15,5)
(20,15,20)	(30,15,25)	(25,15,30)
(30,15,25)	(10,15,5)	(0,15,20)
(25,15,30)	(20,15,20)	(5,15,10)
(15,15,15)	(5,15,10)	(10,15,5)
(5,15,10)	(25,15,30)	(15,15,15)
(10,155)	(15,15,15)	(30,15,25)

Clearly H = {(20,15,20), (30,15,25), (25,15,30), (15,15,15), (10,15,5), (5,15,10)} is a classical group under × with (15, 15, 15) as its identity.

However (7, 21, 28) × (10, 15, 5) = (0, 0, 0) and so on.

Next we consider the second table.

×	(7,21,28)	(28,21,7)	(21,21,21)	(14,21,14)
(7,21,28)	(14,21,14)	(21,21,21)	(7,27,28)	(28,21,7)
(28,21,7)	(21,21,21)	(14,21,14)	(28,21,7)	(7,21,28)
(21,21,21)	(7,21,28)	(28,21,7)	(21,21,21)	(14,21,14)
(14,21,14)	(28,21,7)	(7,21,28)	(14,21,14)	(21,21,21)

We see K = {(7, 21, 28), (28, 21, 7), (21, 21, 21), (14, 21,14)} forms a classical group under product with (21, 21, 21) as its multiplicative identity.

Further
$(7, 21, 28) \times (7, 21, 28) \times (7, 21, 28) \times (7, 21, 28) = (21, 21, 21)$.

That is K is a cyclic group of order four generated by $(7, 21, 28)$ as $(7, 21, 28)^4 = (21, 21, 21)$.

Further $(5, 15, 10)^6 = (15, 15, 15)$ thus H is generated by $(5, 15, 10)$ as a cyclic group of order six.

However $K \times H = (0, 0, 0)$.

Thus the group K annihilates H and vice versa.

However $\langle K \cup H\rangle$ generates a semigroup with (0, 0, 0) as included element of the generating set $\langle K \cup H\rangle$ under product.

Example 1.9. Let $S = \{Z_{33}, \times\}$ be the semigroup under $\times$. 12 and 22 are the idempotents in S. (3, 12, 15) and (15, 12, 3) are neutrosophic triplets associated with the idempotent 12.

(6, 12, 24) and (24, 12, 6) are neutrosophic triplets.

(9, 12, 27) and (27, 12, 9) are both neutrosophic triplets.

(11, 22, 11) is a neutrosophic triplet.

(18, 22, 30) and (30, 12, 18) are both neutrosophic triplets associated with the neutral element 22.

(21, 12, 21) is again a neutrosophic triplet.

(12, 12, 12) and (22, 22, 22) are both neutrosophic triplets.

H = {(3, 12, 15), (15, 12, 3), (6, 12, 24), (24, 12, 6), (9, 12, 27), (27, 12, 9), (18, 12, 30), (30, 12, 18), (21, 12, 21), (12, 12, 12)} is a group under product with (12, 12, 12) as the multiplicative identity.

Clearly $(30, 12, 18)^{10} = (12, 12, 12)$ so H is a cyclic group of order 10 generated by (30, 12, 18).

Now {(11, 22, 11), (22, 22, 22)} = K is also a cyclic group of order two with $(11, 22, 11)^2 = (22, 22, 22)$ so (22, 22, 22) is the identity element of K.

Further K × H = {(0, 0, 0)}. Thus it is assumed that for S = $\{Z_{3p}, \times\}$, p an odd prime under product modulo 3p. S has neutrosophic triplets which are p + 1 in number and that p + 1 neutrosophic triplets can be divided two groups of order p – 1 and 2 and both are cyclic groups with (p + 1, p + 1, p + 1) as identity in case of the group of order p – 1 and (2p, 2p, 2p) as identity for the group of order two.

The elements of cyclic group of order two are {(p, 2p, p), (2p, 2p, 2p)}.

In certain cases for 3p, p + 1 and 2p are idempotents in some cases p and 2p + 1 are idempotents.

First we discuss a few cases to this effect.

We see in case of Z_{15}; 6 and 10 are the idempotents 6 = (5 + 1) and 10 = 5 + 5.

In case of Z_{21} we see 7 and 15 are the idempotents we see 7 and 15 = (2 × 7 + 1).

In case of Z_{33} we see 12 and 22 are the idempotents and 12 = 11 + 1 and 22 = 2 × 11.

In case of Z_{39} we see 13 and 27 are idempotents and 27 = (2 × 13) + 1.

In case of Z_{51} the idempotents are 18 and 34 are idempotents of Z_{51}, 18 = 17 + 1 and 34 = 2 × 17.

Consider Z_{57}, the idempotents are 19 and 39 = (2 × 19) + 1.

Now for Z_{69} we see the idempotents are 24 and 46.

For Z_{87} 30 = 29 + 1 and 58 = 2 × 29 are the only idempotents of Z_{69}.

Consider Z_{159}, the idempotents of Z_{159} are 54 = 53 + 1 and 106 = 2 × 53.

Thus we see in case of Z_{3p} p an odd prime the idempotents of Z_{3p} are either p and 2p + 1 or p + 1 and 2p. Thus we leave it as an open problem the following conjecture.

Conjecture 1.1. Let S = $\{Z_{3p}, \times\}$ be the semigroup under product modulo 3p where p is an odd prime ($p \neq 3$).

i) Characterize those numbers 3p for which p and 2p + 1 are the only nontrivial idempotents of S = $\{Z_{3p}, \times\}$.

ii) Characterize those numbers 3p for which p + 1 and 2p are the only nontrivial idempotents.

Table of the idempotents in Z_{3p}

S. No.	Z_{3p}	p	p + 1	2p	2p + 1
1	Z_{15}	-	6	10	-
2	Z_{21}	7	-	-	15
3	Z_{33}	-	12	22	-
4	Z_{39}	13	-	-	27
5	Z_{51}	-	18	34	-
7	Z_{57}	19	-	-	39
8	Z_{69}	-	24	46	-
9	Z_{159}	-	54	106	-

We however wish to make the following observation and propose the second conjecture.

Conjecture 1.2. Let $\{Z_{3p}, \times\}$, p an odd prime, $p \neq 3$ be the semigroup under product modulo 3p.

i) The sum of digits of 3p when added is either 3 or 6.

ii) If the sum of the digits of 3p is 3 then p and 2p + 1 in Z_{3p} are the only idempotents.

iii) If the sum of the digits of 3p is 6 then p + 1 and 2p are the only idempotents of Z_{3p}.

We just illustrate this by some examples.

In Z_{15}, we see 1 + 5 = 6 so p + 1 = 6 = 5 + 1 and 2p = 10 are the idempotents.

In Z_{219}, we see 2 + 1 + 9 = 3 so p = 73 and 2p + 1 = (2 × 73) + 1 = 147 are the idempotents.

Interested reader can verify for other values of Z_{3p} (p ≠ 3 and p an odd prime).

Next we proceed onto describe by examples semigroups built using Z_{4p}, p an odd prime.

Example 1.10. Let $\{Z_{12}, \times\}$ be the semigroup under product modulo 12. 9 and 4 are the idempotents of S.

Example 1.11. Let S = $\{Z_{20}, \times\}$ be the semigroup under product modulo 20. 5 and 16 are idempoents of Z_{20}.

Example 1.12. Let S = $\{Z_{28}, \times\}$ be the semigroup under product 8 × 8 = 8 (mod 28) (2 × 4 = 8).

21 × 21 = 21 (mod 28) (3 × 7 = 21) are the only idempotents.

Example 1.13. Let S = $\{Z_{44}, \times\}$ be the semigroup under product 12 is an idempotent;

12 = 11 + 1 and 33 = 3 × 11.

Example 1.14. Let S = $\{Z_{52}, \times\}$ be the semigroup under product modulo 52, 13 is an idempotent and 40 = 3 × 13 + 1 is an idempotent.

Example 1.15. Let S = $\{Z_{76}, \times\}$ be the semigroup under product modulo 76.

20 = 19 + 1 is an idempotent.

57 = 3 $\times$ 19 is another idempotent of S.

Example 1.16. Let S = $\{Z_{212}, \times\}$ be the semigroup under product. p = 53 is an idempotent and 160 = (3 $\times$ 53) +1 is an idempotent of S.

Example 1.17. Let S = $\{Z_{388}, \times\}$ be the semigroup under product p = 97 is an idempotent of S. 292 = 3 $\times$ 97 + 1 is an idempotent.

Example 1.18. Let S = $\{Z_{332}, \times\}$ be the semigroup under product modulo 332.

84 = 83 + 1 is an idempotent of S.

249 = 3 $\times$ 83 is another idempotent of S.

We now table the idempotents to find the form of idempotents of Z_{4p}; viz. p + 1 and 3p or p and 3p + 1.

Here p takes prime values which are odd. Further we see there are only two idempotents which is described by the following table.

S. No.	Z_{4p}	p	p + 1	3p	3p+1
1	Z_{12}	-	4	9	-
2	Z_{20}	5	-	-	16
3	Z_{28}	-	8	21	-
4	Z_{44}	-	12	33	-
5	Z_{52}	13	-	-	40
6	Z_{76}	-	20	57	-
7	Z_{212}	53	-	-	160
8	Z_{388}	97	-	-	292
9	Z_{332}	-	84	249	-

In view of this we put forth the following conjecture.

Conjecture 1.3. Let S = $\{Z_{4p}, \times\}$ be the semigroup under product modulo 4p, p an odd prime.

The idempotents of S are either of the form p + 1 and 3p or of the form p and 3p + 1, prove.

Next we study those neutrosophic triplets associated with Z_{2pq} where p and q are odd primes p ≠ q, by some examples.

Example 1.19. Let S = $\{Z_{42}, \times\}$ be the semigroup under product modulo 42.

The idempotents of Z_{42} are 7, 15, 21, 28 and 36.

(2, 22, 32) and (32, 22, 2) are neutrosophic triplets.

(3, 15, 33) and (33, 15, 3) are neutrosophic triplets.

(4, 22, 16) and (16, 22, 4) are neutrosophic triplets.

(8, 22, 8) is a neutrosophic triplet.

(9, 15, 39) and (39, 15, 9) are neutrosophic triplets.

(10, 22, 40) and (40, 22, 10) are neutrosophic triplets.

(12, 36, 24) and (24, 36, 12) are neutrosophic triplets of Z_{42}.

(18, 36, 30) and (30, 36, 18) are neutrosophic triplets.

(20, 22, 20) is a neutrosophic triplet.

(38, 22, 26) and (26, 22, 38) are neutrosophic triplets.

(34, 22, 34) is a neutrosophic triplet.

(27, 15, 27) is a neutrosophic triplet. (6, 36, 6) is a neutrosophic triplet.

(35, 7, 35) is a neutrosophic triplet.

(14, 28, 14) is a neutrosophic triplet.

This Z_{42}, $42 = 2 \times 3 \times 7$ behaves in a very different way.

In the first place Z_{42} has idempotents under product modulo 42. They are 7, 15, 21, 22, 28 and 36.

Further (7, 7, 7), (15, 15, 15), (21, 21, 21), (22, 22, 22), (28, 28, 28) and (36, 36, 36) are also neutrosophic triplets.

There are 29 nontrivial neutrosophic triplets. (0, 0, 0) is taken as the trivial neutrosophic triplet.

Now these 29 neutrosophic triplets forms a semigroup of order 29. However the 28 triplet baring the trivial (0, 0, 0) triplet

is not even closed under product as (3, 15, 33) × (14, 28, 14) = (42 (mod 42), 420 (mod 42), 462 (mod 42)) = (0, 0, 0). Hence this is only a semigroup.

Now we collect the neutrosophic triplets which has 7 as the neutral element.

K_1 = {(7, 7, 7), (35, 7, 35)} is a neutrosophic group of order two given by the following table.

×	(7,7,7)	(35,7,35)
(7, 7, 7)	(7, 7, 7)	(35, 7, 35)
(35, 7, 35)	(35, 7, 35)	(7, 7, 7)

So K_1 is a cyclic group of order two with (7,7,7) as the multiplicative identity.

Let K_2 = {(15, 15, 15), (3, 15, 33), (33, 15, 3), (9, 15, 39), (39, 15, 9), (27, 15, 27)} be a group under × associated with the neutral element 15.

The table of K_2 is group of order 6 which is as follows.

×	(3,15,33)	(33,15,3)	(15,15,15)
(3,15,33)	(9,15,39)	(15,15,15)	(3,15,33)
(33,15,3)	(15,15,15)	(39,15,9)	(33,15,3)
(15,15,15)	(3,15,33)	(35,15,3)	(15,15,15)
(9,15,39)	(27,15,27)	(30,15,33)	(9,15,39)
(39,15,9)	(33,15,3)	(27,15,27)	(39,15,9)
(27,15,27)	(39,15,9)	(9,15,39)	(27,15,27)

(9,15,39)	(39,15,9)	(27,15,27)
(27,15,27)	(33,15,3)	(39,15,9)
(30,15,33)	(27,15,27)	(9,15,39)
(9,15,39)	(39,15,9)	(27,15,27)
(39,15,9)	(15,15,15)	(33,15,3)
(15,15,15)	(9,15,39)	(3,15,33)
(33,15,3)	(3,15,33)	(15,15,15)

We see (3,15,33) generates K_2 as a cyclic group of order 6. Thus K_2 is a cyclic group with (15, 15, 15) as the identity. That is $(3, 15, 33)^6 = (15, 15, 15)$.

Consider K_3 = {(21, 21, 21)} this an neutrosophic triplet which is such that (7, 7, 7) × (21, 21, 21) = (21, 21, 21), (15, 15, 15) × (21, 21, 21) = (21, 21, 21), (22, 22, 22) × (21, 21, 21) = (0, 0, 0), (28, 28, 28) × (21, 21, 21) = (0, 0, 0) and (36, 36, 36) × (21, 21, 21) = (0, 0, 0).

Thus the neutrosophic triplet groups behaves uniquely for it does not neutral any of the non units of Z_{42}.

Let K_4 = {(2, 22, 32), (32, 22, 2), (4, 22 16), (16, 22, 4), (22, 22, 22), (8, 22, 8), (10, 22, 40), (40, 22, 10) (20, 22, 20) (26, 22, 38), (38 22, 26), (34, 22, 34)} be a group with the neutral element 22.

The table for K_4 is as follows.

×	(2, 22, 32)	(32, 22, 2)	(4, 22, 16)
(2, 22, 32)	(4, 22, 16)	(22, 22, 22)	(8, 22, 8)
(32, 22, 2)	(22, 22, 22)	(16,22,4)	(2,22,32)
(4,22,16)	(8,22,8)	(2,22,32)	(16,22,4)
(16,22,4)	(32,22,2)	(8,22,8)	(22,22,22)
(22,22,22)	(2,22,32)	(32,22,2)	(4,22,16)
(8,22,8)	(16,22,4)	(4,22,16)	(32,22,2)
(10,22,40)	(20,22,20)	(26,22,38)	(40,22,10)
(40,22,10)	(38,22,26)	(20,22,20)	(34,22,34)
(20,22,20)	(40,22,10)	(10,22,40)	(38,22,26)
(26,22,38)	(10,22,40)	(34,22,34)	(20,22,20)
(38,22,26)	(34,22,34)	(40,22,10)	(26,22,38)
(34,22,34)	(26,22,38)	(38,22,26)	(10,22,40)

(16, 22, 4)	(22, 22, 22)	(8,22,8)	(10,22,40)	(40,22,10)
(32, 22, 2)	(2, 22, 32)	(16,22,4)	(20,22,30)	(38,22,26)
(8,22,8)	(32,22,2)	(4,22,16)	(26,22,38)	(20,22,20)
(22,22,22)	(4,22,16)	(32,22,2)	(40,22,10)	(34,22,34)
(4,22,16)	(16,22,4)	(2,22,32)	(34,22,34)	(10,22,40)
(16,22,4)	(22,22,22)	(8,22,8)	(10,22,40)	(40,22,10)
(2,22,32)	(8,22,8)	(22,22,22)	(38,22,26)	(26,22,38)
(34,22,34)	(10,22,40)	(38,22,26)	(16,22,4)	(22,22,22)
(10,22,40)	(40,22,10)	(26,22,38)	(22,22,22)	(4,22,16)
(26,22,38)	(20,22,20)	(34,22,34)	(32,22,2)	(2,22,32)
(38,22,26)	(38,22,26)	(40,22,10)	(8,22,8)	(32,22,2)

(20,22,20)	(38,22,26)	(10,22,40)	(2,22,32)	(8,22,8)
(40,22,10)	(34,22,34)	(20,22,20)	(4,22,16)	(16,22,4)

(20,22,20)	(26,22,38)	(38,22,26)	(34,22,34)
(40,22,10)	(10,22,40)	(34,22,34)	(26,22,38)
(10,22,40)	(34,22,34)	(40,22,10)	(38,22,26)
(38,22,26)	(20,22,20)	(26,22,38)	(10,22,40)
(26,22,38)	(38,22,26)	(20,22,20)	(40,22,10)
(20,22,20)	(26,22,38)	(38,22,26)	(34,22,34)
(34,22,34)	(40,22,10)	(10,22,40)	(20,22,20)
(32,22,2)	(8,22,8)	(2,22,32)	(4,22,16)
(2,22,32)	(32,22,2)	(8,22,8)	(16,22,4)
(40,22,10)	(16,22,4)	(4,22,16)	(8,22,8)
(16,22,4)	(4,22,16)	(22,22,22)	(2,22,32)
(4,22,16)	(22,22,22)	(16,22,4)	(32,22,2)
(8,22,8)	(2,22,32)	(32,22,2)	(22,22,22)

This is a group of order 12 with (22, 22, 22) as the identity element.

Clearly K_4 is a cyclic group.

Next consider K_5 = ({14, 28, 14), (28, 28, 28)} is a cyclic subgroup of order 2 with 28 as the neutral element and (28, 28, 28) as the identity element of K_5.

Let K_6 = {(12, 36, 24), (6, 36, 6), (24, 36, 12), (30, 36, 18), (18, 36, 30), (36, 36, 36)} be a group of order 6 with 36 as the neutral element. The table of K_6 is as follows.

×	(6, 36, 6)	(12, 36, 24)
(6, 36, 6)	(36, 36, 36)	(30, 36, 18)
(12, 36, 24)	(30, 36, 18)	(18, 36, 30)
(24, 36, 12)	(18, 36, 30)	(36, 36, 36)
(36, 36, 36)	(6, 36, 6)	(12, 36, 24)
(30, 36, 18)	(12, 36, 24)	(24, 36, 12)
(18, 36, 30)	(24, 36, 12)	(6, 36, 6)

(24,36,12)	(36,36,36)	(30,36,18)	(18,36,30)
(18,36,30)	(6,36,6)	(12,36,24)	(24,36,12)
(36,36,36	(12,36,24)	(24,36,12)	(6,36,6)
(30,36,18)	(24,36,12)	(6,36,6)	(12,36,24)
(24,36,12)	(36,36,36)	(30,36,18)	(18,36,30)
(6,36,6)	(30,36,18)	(18,36,30)	(36,36,36)
(12,36,24)	(18,36,30)	(36,36,36)	(30,36,18)

It is easily verified this is a cyclic group; for

$$
\begin{aligned}
(6, 36, 6)^2 &= (36, 36, 36) \\
(12, 36, 24)^2 &= (18, 36, 30) \\
(12, 36, 24)^3 &= (18, 36, 30) \times (12, 26, 24) \\
&= (6, 36, 6) \\
(12, 36, 24)^4 &= (6, 36, 6) \times (12, 36, 24) = (30, 36, 18) \\
(12,36,24)^5 &= (30, 36, 18) \times (12, 36, 24) \\
&= (24, 36, 12) \\
\text{and} \quad (12, 36,24)^6 &= (24,36,12) \times (12,36,24) \\
&= (36, 36, 36).
\end{aligned}
$$

Thus K_6 is as a cyclic group generated by (12,36,24).

Thus we get several neutral elements and several neutrosophic triplets.

We will see the probable applications of them in the last chapter.

Example 1.20. Let $S = \{Z_{66}, \times\}$ be the semigroup under product modulo $66 = 2 \times 3 \times 11$. 12, 22, 33, 34, 45 and 55 are the idempotents of Z_{66}. All these 6 idempotents serve as neutral elements.

(2, 34, 50) and (50, 34, 2) are neutrosophic triplets of $34 \in Z_{66}$.

(4, 34, 58) and (58, 34, 4) are neutrosophic triplets of Z_{66}.

(8, 34, 62) and (62, 34, 8) are neutrosophic triplets.

(16, 34, 64) and (64, 34, 16) neutrosophic triplets.

(32, 34, 32) is a neutrosophic triplet.

(3, 45, 15) and (15, 45, 3) are neutrosophic triplets.

(9, 45, 27) and (27, 45, 9) are neutrosophic triplets.

(10, 34, 10) is a neutrosophic triplet.

(20, 34, 38) and (38, 34, 20) are neutrosophic triplets.

(40, 34, 52) and (52, 34, 40) are neutrosophic triplets.

(14, 34, 26) and (26 34, 14) are neutrosophic triplets.

(28, 34, 46) and (46, 34, 28) are neutrosophic triplets.

` (56, 34, 56) is a neutrosophic triplet.

(30, 12, 18) and (18, 22, 30) are neutrosophic triplets.

(42, 12,60) and (60, 12, 42) are neutrosophic triplets.

(6, 12, 24) and (24, 12, 6) are neutrosophic triplets.

(48, 12, 36) and (36, 12, 48) are neutrosophic triplets.

(54, 12, 54) is a neutrosophic triplet.

(11, 33, 21) and (21, 33, 11) are neutrosophic triplets.

(45, 33, 45) is a neutrosophic triplet.

(39, 45, 57) and (57, 45, 39) are neutrosophic triplets.

(44, 22, 44) is a neutrosophic triplet.

(51, 45, 63) and (63, 45, 51) is a neutrosophic triplets.

We have (12, 12, 12), (22, 22, 22), (33, 33, 33), (34, 34, 34), (45, 45, 45) and (55, 55, 55) to be neutrosophic triplets which either act as identity under product or act as annihilator under product.

We see (22, 22, 22) × (51, 45, 63) = (0, 0, 0).

(34, 34, 34) × (58, 34, 4) = (58, 34, 4) and so on. We see there are 47 such nontrivial neutrosophic triplets.

We will check which of them form a group under × and the order of them.

Let K_1 be the collection of all those neutrosophic triplets which has 12 as its neutral element.

K_1 = {(12, 12, 12) (30, 12, 18), (18, 12 30), (42, 12, 60), (60, 12, 42), (24, 12, 6), (6, 12, 24), (48, 12, 36), (36, 12, 48), (54, 14, 54)}.

The table for K_1 is as follows: (12, 12, 12) acts as the multiplicative identity of K_1. We test whether the group is cyclic or not.

$\times$	(12, 12, 12)	(30,12,18)
(12,12,12)	(12,12,12)	(30,12,18)
(30,12,18)	(30,12,18)	(42,12,60)
(18,12,30)	(18,12,30)	(12,12,12)
(42,12,60)	(44,12,60)	(6,12,24)
(60,12,42)	(60,12,42)	(18,12,30)
(24,12,6)	(24,12,6)	(60,12,42)
(6,12,24)	(6,12,24)	(48,12,36)
(48,12,36)	(48,12,36)	(54,12,54)
(36,12,48)	(36,12,48)	(24,12,6)
(54,12,54)	(54,12,54)	(36,12,48)

(18,12,30)	(42,12,60)	(60,12,42)
(18,12,30)	(42,12,60)	(60,12,42)
(12,12,12)	(6,12,24)	(18,12,30)
(60,12,42)	(30,12,18)	(24,12,6)
(30,12,18)	(48,12,36)	(12,12,12)
(24,12,6)	(12,12,12)	(36,12,48)
(36,12,48)	(18,12,30)	(54,12,54)
(42,12,60)	(54,12,54)	(30,12,18)
(6,12,24)	(36,12,48)	(24,12,6)
(54,12,54)	(60,12,42)	(48,12,36)
(48,12,36)	(24,12,6)	(6,12,24)

(24,12,6)	(6,12,24)	(48,12,36)
(24,12,6)	(6,12,24)	(48,12,36)
(60,12,42)	(48,12,36)	(54,12,54)
(36,12,48)	(42,12,60)	(6,12,24)
(18,12,30)	(54,12,54)	(36,12,48)
(54,12,54)	(30,12,18)	(24,12,6)
(48,12,36)	(12,12,12)	(30,12,18)
(12,12,12)	(36,12,48)	(24,12,6)
(30,12,18)	(24,12,6)	(60,12,42)
(6,12,24)	(18,12,36)	(12,12,12)
(42,12,60)	(60,12,42)	(18,12,36)

(36,12,48)	(54,12,54)
(36,12,48)	(54,12,54)
(24,12,6)	(36,12,48)
(54,12,54)	(48,12,36)
(60,12,42)	(24,12,6)
(48,12,36)	(6,12,24)
(6,12,24)	(42,12,60)
(18,12,36)	(60,12,42)
(12,12,12)	(18,12,36)
(42,12,60)	(30,12,18)
(30,12,18)	(12,12,12)

Clearly (30, 12, 18) generates K_1 as a cyclic group of order 10 as

$$(30, 12, 18)^{10} = (12, 12, 12).$$

Let $K_2 = \{(44, 22, 44), (22, 22, 22)\}$

be the cyclic group of order 2 with 22 as the neutral element and (22, 22, 22) as the group identity.

Let $K_3 = \{(11, 33, 21), (21, 33, 11), (33, 33, 33), (55, 33, 45), (45, 33, 55)\}$.

The table for K_3 is as follows.

×	(11,33,21)	(21,33,11)	(33,33,33)	(45,33,55)	(55,33,45)
(11,33,21)	(55,33,45)	(33,33,33)	(11,33,21)	(33,33,33)	(11,33,21)
(21,33,11)	(33,33,33)	(45,33,55)	(21,33,11)	(21,33,11)	(33,33,33)
(33,33,33)	(11,33,21)	(21,33,11)	(33,33,33)	(45,33,55)	(55,33,45)
45,33,55)	(33,33,33)	(21,33,11)	(45,33,55)	(45,33,55)	(33,33,33)
(55,33,45)	(11,33,21)	(33,33,33)	(55,33,45)	(33,33,33)	(55,33,45)

Clearly K_3 is not a group only a semigroup under product.

Consider K_4 = {(34, 34, 34), (2, 34, 50), (50, 34, 2), (4, 34, 58), (58, 34, 4), (8, 34, 62), (62, 34, 8), (16, 34, 64), (64, 34, 16), (32, 34, 32), (10, 34, 10), (20, 34, 38), (38, 34, 20), (40, 34, 52), (52, 34, 40), (14, 34, 26), (26, 34, 14), (28,34,46) (46, 34,28), (56,34,56)}.

The reader is left with the task of finding whether the collection in K_4 is a group or not. Finally we have collection with 45 as the neutral element.

K_5 = {(3, 45, 15), (15, 45, 3), (45, 45, 45), (21, 45, 21), (9, 45, 27), (27, 45, 9), (39, 45, 57), (51, 45, 63), (63, 45, 51), (57, 45, 39)}.

We find the table under product.

×	(3,45,15)	(15,45,3)	(45,45,45)
(3,45,15)	(9,45,27)	(45,45,45)	(3,45,15)
(15,45,3)	(45,45,45)	(27,47,9)	(15,45,3)
(45,45,45)	(3,45,15)	(15,45,3)	(45,45,45)
(9,45,27)	(27,45,9)	(3,45,15)	(9,45,27)
(27,45,9)	(15,45,3)	(9,45,27)	(27,45,9)
(39,45,57)	(51,45,63)	(57,45,39)	(39,45,57)
(57,45,39)	(39,45,57)	(63,45,51)	(57,45,39)
(51,45,63)	(21,45,21)	(39,45,57)	(51,45,63)
(63,45,51)	(57,45,39)	(21,45,21)	(63,45,51)
(21,45,21)	(63,45,51)	(51,45,63)	(21,45,21)

(9,45,27)	(27,45,9)	(21,45,21)	(39,45,57)
(27,45,9)	(15,45,3)	(63,45,51)	(51,45,63)
(3,45,15)	(9,45,27)	(51,45,63)	(57,45,39)
(9,45,27)	(27,45,9)	(21,45,21)	(39,45,57)
(15,45,3)	(45,45,45)	(57,45.39)	(21,45,21)
(9,45,45)	(3,45,15)	(39,45,57)	(63,45,51)
(21,45,21)	(63,45,51)	(27,45,9)	(3,45,15)
(51,45,63)	(21,45,21)	(9,45,27)	(45,45,45)
(63,45,51)	(57,45,39)	(15,45,3)	(9,45,27)
(39,45,57)	(51,45,63)	(3,45,15)	(15,45,3)
(57,45,39)	(39,45,57)	(45,45,45)	(27,45,9)

(57,45,39)	(51,45,63)	(63,45,51)
(39,45,57)	(21,45,21)	(57,43,39)
(63,45,51)	(39,45,57)	(21,45,21)
(57,45,39)	(51,45,63)	(63,45,51)
(51,45,63)	(63,45,51)	(39,45,57)
(21,45,21)	(57,45,39)	(51,45,63)
(45,45,45)	(9,45,27)	(15,45,3)
(15,45,3)	(3,45,15)	(27,45,9)
(3,45,15)	(27,45,9)	(45,45,45)
(27,45,9)	(45,45,45)	(9,45,27)
(9,45,27)	(15,45,3)	(3,45,15)

We see K_5 is a group of order 10 with 45 as the neutral element and (45, 45, 45) as the identity element.

Clearly K_5 is a cyclic group of order 10 and (51, 45, 63) generates K_5 that is $(51, 45, 63)^{10} = (45, 45, 45)$. (55, 55, 55) is a neutrosophic triplet and there are no nontrivial triplets with 55 as a neutral element.

Thus we see some neutral elements yield the collection of the neutrosophic triplets associated with that neutral element to be a cyclic group whereas others result in semigroups.

Here the neutral element 33 yields only a semigroup. Study in this direction is interesting and innovative.

Example 1.21. Let S = $\{Z_{36}, \times\}$ be the semigroup under product modulo 36. The only idempotents of 36 are 9 and 28. (4, 28, 16) and (16, 28, 4) are neutrosophic triplets. (8, 28, 8) is a neutrosophic triplet. (28, 28, 28) is also a neutrosophic triplet.

Further (32, 28, 20) and (20, 28, 32) are neutrosophic triplets.

(21, 9, 21) is a neutrosophic triplet.

However it is difficult to find all neutrosophic triplets.

So the task finding all idempotents and neutrosophic triplets for elements in Z_n where $n = 4.3^2 = 2^2\ 3^2$ or in general $n = 2^2p$ where p is an odd prime happens to be a very difficult problem.

Next we proceed onto give a open conjecture in this regard.

Conjecture 1.4. Let S = $\{Z_n, \times\}$ where $n = 2^2p^2$ where p is an odd prime be the semigroup under product modulo 2^2p^2.

i) Can S have more than two idempotents?
ii) Find all nontrivial idempotents of 2^2p^2.
iii) Are they of the p^2 and $4 \times q$ where q is a largest prime in $Z_{p^2 2^2}$ such that $4q < p^2 2^2$?

Next we proceed onto describe some more Z_n for different n and derive some relations.

Example 1.22. Consider S = $\{Z_{105}, \times\}$ the semigroup under product modulo 105. 15, 21, 36, 70 and 91 are idempotents of 105.

(3, 36, 12), (9, 36, 39), (27, 36, 48),

(81, 36, 51), (33, 36, 87), (99, 36, 99),

(6, 36, 6), (18, 36, 72), (54, 36, 24),

(57, 36, 78), (66, 36, 96), (93, 36, 102),

(69, 36, 69), (102, 36, 93), (96, 36, 66),

(78, 36, 57), (24, 36, 54), (72, 36, 18),

(87, 36, 33), (51, 36, 81), (48, 36, 27),

(39, 36, 9), (12, 36, 3), (7, 91, 28), (49, 91, 49), (28, 91, 7)

(14,91,14), (91,91,91), (98,91,77), (77,91,98), (35,70,35)

(70,70,70), (7,70,25), (49,70,100) etc. are some of the neutrosophic triplet groups.

We see the elements which contribute neutrosophic triplets happens to be a difficult job. However there are 5 idempotents in Z_{105}, $105 = 3 \times 5 \times 7$.

We see finding idempotents and the related neutrosophic triplets happens to be difficult job if n = pqr where p, q and r are three distinct odd primes.

Even in case of $n = 105 = 3 \times 5 \times 7$ we find it difficult to find the neutrosophic triplets and the corresponding algebraic structure built on them.

Example 1.23. Let $S = \{Z_{165}, \times\}$ be the semigroup under product modulo 165 = 3.5.11. $45 \in Z_{165}$ is an idempotent, $45 \times 45 = 45 \pmod{165}$;

$55 \times 55 = 55 \pmod{165}$, $66 \times 66 = 66 \pmod{165}$,

$100 \times 100 = 100 \pmod{165}$, $111 \times 111 = 111 \pmod{165}$,

$121 \times 121 = 21 \pmod{165}$ and $130 \times 130 = 130 \pmod{165}$.

(3, 111, 147) and (147, 111, 3) are neutrosophic triplets of S.

(9, 111, 159) and (159, 111, 9) are neutrosophic triplets of S.

(27, 111, 108) and (108, 111, 27) are neutrosophic triplets.

(81, 111, 36) and (36, 111, 81) are neutrosophic triplets.

(111, 111, 111) is a neutrosophic triplet.

(10, 55, 22) and (22, 55, 10) are neutrosophic triplets.

(100, 55, 154) and (154, 55, 100) are neutrosophic triplets.

(11, 66, 6) and (6, 66, 11) are neutrosophic triplets.

(66, 66, 66), (36, 66, 121) and (121, 66, 36) are neutrosophic triplets.

(51, 66, 11) and (11, 66, 51) are neutrosophic triplets.

(141, 66, 121) and (121, 66, 141), (12, 111, 78)

and (78, 111, 12) are neutrosophic triplets.

(144, 111, 144) is a neutrosophic triplet.

(21, 111, 21) is a neutrosophic triplet.

(24, 111, 39) and (39, 111, 24) are neutrosophic triplets.

We find it difficult to get all neutrosophic triplets.

We propose the following conjecture.

Conjecture 1.5. Let $S = \{Z_n, \times\}$ where n = pqr where p, q and r are three distinct odd primes.

i) Find the number of idempotents in Z_n.
ii) Are these idempotents dependent on p, q and r?
iii) Find the total number of neutrosophic triplets associated with Z_n.
iv) Does these number in (iii) dependent on n?

Example 1.24. Let $S = \{Z_n, \times\}$ where n = 2.3.5.7 = 210 be a semigroup under product modulo 210.

$15 \times 15 = 225 \equiv 15 \pmod{210}$, $21 \times 21 \equiv 21 \pmod{210}$,

$36 \times 36 \equiv 36 \pmod{210}$, $70 \times 70 \equiv 70 \pmod{210}$,

$85 \times 85 \equiv 85 \pmod{210}$, $91 \times 91 \equiv 91 \pmod{210}$,

$106 \times 106 \equiv 106 \pmod{210}$, $105 \times 105 \equiv 105 \pmod{210}$,

$120 \times 120 \equiv 120 \pmod{210}$, $126 \times 126 \equiv 126 \pmod{210}$,

$141 \times 141 \equiv 141 \pmod{210}$, $175 \times 175 \equiv 175 \pmod{210}$,

$190 \times 190 \equiv 190 \pmod{210}$, $196 \times 196 \equiv 196 \pmod{210}$,

and $200 \times 200 \equiv 200 \pmod{210}$.

We see there are 15 idempotents in Z_{210}.

But $o(Z_{210}) = 210$.

We just give a few neutrosophic triplets. We see if in Z_n, n is a product of more number of primes and it also includes 2 as a prime there are always a chance of getting more number of idempotents and neutrosophic triplets.

(2, 106, 158) and (158, 106, 2) are neutrosophic triplets.

(4, 106, 184) and (184, 106, 4) are neutrosophic triplets.

(8, 106, 92) and (92, 106, 8) are neutrosophic triplets.

(16, 106, 46) and (46, 106, 16) are neutrosophic triplets.

(32, 106, 128) and (128, 106, 32) are neutrosophic triplets.

(64, 106, 84) and (84, 106, 64) are neutrosophic triplets.

(3, 141, 117) and (117, 141, 3) are neutrosophic triplets.

(9, 141, 39) and (39, 141, 9) are neutrosophic triplets.

(27, 141, 153) and (153, 141, 27) are neutrosophic triplets.

(81, 141, 51) and (51, 141, 81) are neutrosophic triplets.

(33, 141, 97) and (87, 141, 33) are neutrosophic triplets.

(99, 141, 117) and (117, 141, 99) are neutrosophic triplets.

(5, 85, 185) and (185, 85, 5) are neutrosophic triplets.

(25, 85, 205) and (205, 85, 25) are neutrosophic triplets.

(125, 85, 125) is a neutrosophic triplet.

(7, 70, 40) and (40, 70, 7) are neutrosophic triplets.

(49, 70, 130) and (130, 70, 49) are neutrosophic triplets.

(133, 70, 160) and (160, 70, 133) are neutrosophic triplets.

(6, 36, 6) and (36, 36, 36) are neutrosophic triplets.

(8, 106, 92) and (92, 106, 8) are neutrosophic triplets.

(64, 106, 64) and (106, 106, 106) are neutrosophic triplets.

(12, 36, 108) and (108, 36, 12) are neutrosophic triplets.

(72, 36, 18) and (18, 36, 72) are neutrosophic triplets.

However even for this small value as Z_{210} finding all neutrosophic triplets happens to be a very challenging problem.

We conjecture the following.

Conjecture 1.6. Let $\{Z_n, \times\} = S$ be the semigroup under product modulo n.

Can we say if n is even S has more number of neutrosophic triplets and idempotents?

Conjecture 1.7. Let $S_1 = \{ Z_{m_1}, \times\}$ where $m_1 = 2.p_1\, p_2 \ldots p_t$, p_i's distinct odd primes different from 3.

Let $m_2 = 3p_1p_2\ldots p_t$, distinct odd primes as in m_1 only 2 is replaced by 3, where $S_2 = \{Z_{m_2}, \times\}$.

i) Which of the semigroups S_1 or S_2 has more number of neutrosophic triplets?
ii) Which of S_1 or S_2 has more number of idempotents?
iii) Can we claim only S_2 has more triplets?

We will give one example and conclude this chapter with some problems.

Example 1.25. Let $S_1 = \{Z_{m_1}, \times\}$ where $m_1 = 2.5.7$ and $S_2 = \{Z_{m_2}, \times\}$ where $m_2 = 3.5.7$ be two semigroups under product modulo 70 and 105 respectively.

The idempotents in Z_{70} are

$75 \times 75 = 15 \pmod{70}$, $21 \times 21 = 21 \pmod{70}$

$35 \times 35 = 35 \pmod{70}$,$36 \times 36 = 36 \pmod{70}$

$46 \times 46 = 46 \pmod{70}$, $50 \times 50 = 50 \pmod{70}$ and

$56 \times 56 = 56 \pmod{70}$.

There are 7 idempotents in $Z_{70} = Z_{m_1}$.

Now we find the idempotents of $Z_{105} = Z_{m_2}$.

$15 \times 15 \equiv 15 \pmod{105}$, $21 \times 21 \equiv 21 \pmod{105}$,

$36 \times 36 \equiv 36 \pmod{105}$, $70 \times 70 \equiv 70 \pmod{105}$,

$85 \times 85 \equiv 85 \pmod{105}$ and $91 \times 91 \equiv 91 \pmod{105}$.

We see $Z_{70} = Z_{m_1}$ has 7 idempotents where as $Z_{105} = Z_{m_2}$ has only 6 idempotents.

So the number of neutrosophic triplets in case of Z_{70} will be more than that of Z_{105}. To this effect we work out here.

(2, 36, 18) and (18, 36, 2) are neutrosophic triplets.

(4, 36, 44) and (44, 36, 4) are neutrosophic triplets.

(8, 36, 22) and (22, 36, 8) are neutrosophic triplets.

(16, 36, 46) and (46, 36, 16) are neutrosophic triplets.

(32, 36, 58) and (58, 36, 32) are neutrosophic triplets.

(64, 36, 64) and (36, 36, 36) are neutrosophic triplets.

(5, 15, 45) and (45, 15, 5) are neutrosophic triplets.

(25, 15, 65) and (65, 15, 25) are neutrosophic triplets.

(55, 15, 55) and (15, 15, 15) are neutrosophic triplets and (6, 36, 6) is a neutrosophic triplet.

(7, 21, 63) and (63, 21, 7) are neutrosophic triplets.

(49, 21, 49) and (21, 21, 21) are neutrosophic triplets.

(12, 36, 38) and (38, 36, 12) are neutrosophic triplets.

(14, 56, 14) and (56, 56, 56) are neutrosophic triplets.

(24, 36, 54) and (54, 36, 24) are neutrosophic triplets.

(48, 36, 62) and (62, 36, 48) are neutrosophic triplets.

(66, 36, 26) and (26, 36, 66) are neutrosophic triplets.

(52, 36, 68) and (68, 36, 52) are neutrosophic triplets.

(34, 36, 34) is a neutrosophic triplet.

(40, 50, 10) and (10, 50, 40) are neutrosophic triplets.

(30, 50, 60) and (60, 50, 30) are neutrosophic triplets.

(20, 50, 20) and (50, 50, 50) are neutrosophic triplets.

We have given over 40 such neutrosophic triplets. Finding all of them happens to be a difficult one.

Now we find the neutrosophic triplets associated with Z_{105}.

(3, 36, 12) and (12, 36, 3) are neutrosophic triplets.

(9, 36, 39) and (39, 36, 9) are neutrosophic triplets.

(27, 36, 48) and (48, 36, 27) are neutrosophic triplets.

(81, 36,51) and (51, 36, 81) are neutrosophic triplets.

(33, 36, 87) and (87, 36, 33) are neutrosophic triplets.

(99, 36, 99) and (36, 36, 36) are neutrosophic triplets.

(6, 36, 6) is a neutrosophic triplet.

(18, 36, 72) and (72, 36, 18) are neutrosophic triplets.

(54, 36, 24) and (24, 36, 54) are neutrosophic triplets.

(57, 36, 78) and (78, 36, 57) are neutrosophic triplets.

(66, 36, 96) and (96, 36, 66) are neutrosophic triplets.

(93, 36, 102) and (102, 36, 93) are neutrosophic triplets.

(69, 36, 69) is a neutrosophic triplet.

(7, 91, 28) and (28, 91, 7) are neutrosophic triplets.

(49, 91, 49) and (91, 91, 91) are neutrosophic triplets.

(14, 91, 14) is a neutrosophic triplet.

(98, 91, 77) and (77, 91, 98) are neutrosophic triplets

(56, 91, 56) is a neutrosophic triplet.

(10, 85, 40) and (40, 85, 10) are neutrosophic triplets.

(100, 85, 25) and (25, 85, 100) are neutrosophic triplets.

(55, 85, 55) and (20, 85, 20) are a neutrosophic triplets.

(95, 85, 65) and (65, 85, 95) are neutrosophic triplets.

(5, 85, 80) and (80, 85, 5) are neutrosophic triplets.

(50, 85, 50) is a neutrosophic triplet.

(25, 85, 100) and (100, 85, 25) are neutrosophic triplets.

Thus we see the number can be more in case of Z_{105} than Z_{70}.

Can we make a conclusion the larger the number the more number of neutrosophic triplets.

We see Z_{105} has more neutrosophic triplets than Z_{70}.

Example 1.26. Let $\{Z_{48}, \times\}$ = S be the semigroup under product modulo 48. $48 = 2^4.\ 3$.

The idempotents of Z_{48} are $16 \times 16 \equiv$ (mod 48) is the only one idempotent in S.

(32, 16, 32) and (16, 16, 16) are the only neutrosophic triplets.

So if $n = 2^4p$, p a prime, we can say it has only one nontrivial neutrosophic triplet.

Example 1.27. Let S = $\{Z_{162}, \times\}$ be the semigroup under product modulo 162.

The idempotent in S is $81 \times 81 \equiv$ (mod 162).

Thus we have the following conjecture.

Conjecture 1.8. Let S = $\{Z_n, \times\}$ where $n = p^4q$ where p and q are two different primes then p^4 is the only idempotent of Z_{p^4q}.

We have seen in case of Z_{48} ($48 = 2^4.\ 3$) and Z_{162} ($162 = 3^4.2$) for 16 and 81 are the idempotents respectively.

Thus if we want more idempotents we need to take n in Z_n to be the product of more number of primes.

We give some more examples.

Example 1.28. Let $S = \{Z_{100}, \times\}$ be the semigroup under product modulo 100.

We see the idempotents of Z_{100} are $25 \times 25 = 25 \pmod{100}$

$76 \times 76 \equiv 76 \pmod{100}$ are the only two idempotents.

Thus can we say $2p^2$ where p is an odd prime then Z_n, $n = 2^2p^2$ has only two idempotents.

Example 1.29. Let $S = \{Z_{225}, \times\}$ be the semigroup under product modulo $225 = 9 \times 25 = 3^2 \times 5^2$.

To find all the idempotents associated with Z_{225}

$100 \times 100 = 100 \pmod{225}$ and

$126 \times 126 \equiv 126 \pmod{225}$.

Thus Z_{225} has only two nontrivial idempotents.

Thus we conjecture.

Conjecture 1.9. If $\{Z_n, \times\}$ be a semigroup under product modulo n with $n = p^2\, q^2$, p and q are two distinct primes than Z_n has only two distinct idempotents.

When we work with Z_n, $n = p^3 q^3$ where p and q two distinct primes we study the idempotents in them by some examples.

Example 1.30. Let S = $\{Z_{216}, \times\}$ be the semigroup under product under modulo 216, $216 = 2^3 . 3^3$.

The idempotents of S are $81 \times 81 \equiv 81$ (mod 215) and

$136 \times 136 \equiv 136$ (momd 216).

Hence we face the same problem as that of $Z_{p^2q^2}$ only two idempotents even in case of p^3q^3.

Example 1.31. Let S = $\{Z_{80}, \times\}$ be the semigroup under $\times$ modulo 80.

$16 \times 16 = 16$ (mod 80) and $65 \times 65 \equiv 65$ (mod 80).

S has only two idempotents.

In view of this we just conjecture the following.

Conjecture 1.10. Let S = $\{Z_n, \times\}$ be the semigroup under product where $n = p^t q^s$ where p and q are primes (t and s) are positive integers greater than or equal to 2).

i) Can Z_n have more than two idempotents?
ii) Find all neutrosophic triplets associated with S.
iii) Do they form a group under $\times$?

Example 1.32. Let S = $\{Z_{84}, \times\}$ be a semigroup under product modulo 84.

The idempotents in Z_{84} are

$36 \times 36 \equiv 36 \pmod{84}$, $21 \times 21 \equiv 21 \pmod{84}$

$28 \times 28 \equiv 28 \pmod{84}$, $49 \times 49 \equiv 49 \pmod{84}$

$57 \times 57 \equiv 57 \pmod{84}$ and $64 \times 64 \equiv 64 \pmod{84}$.

(4, 64, 16) and (16, 64, 4) are neutrosophic triplets.

(3, 57, 75) and (75, 57, 3) are neutrosophic triplets.

(9, 57, 81) and (81, 57, 9) are neutrosophic triplets.

(27, 57, 27) and (57, 57, 57) are neutrosophic triplets.

(12, 64, 12) is a neutrosophic triplet.

(48, 64, 24) and (24, 64, 48) are neutrosophic triplets.

(12, 36, 24) and (24, 36, 12) are neutrosophic triplets.

(60, 36, 72) and (72, 36, 60) are neutrosophic triplets.

(48, 36, 48) and (36, 36, 36) are neutrosophic triplets.

(20, 64, 20) is a neutrosophic triplet.

(32, 64, 44) and (44, 64, 32) are neutrosophic triplets.

(52, 64, 40) and (40, 64, 52) are neutrosophic triplets.

(56, 64, 76) and (76, 64, 56) are neutrosophic triplets.

(68, 64, 80) and (80, 64, 68) are neutrosophic triplets.

The reader is left with the task of finding all the neutrosophic triplets.

We see some idempotents gives more neutrosophic triplet groups than others.

There must be some number theoretic arguments related with them.

(7, 49, 7) and (49, 49, 49) are neutrosophic triplets.

(15, 57, 15) is a neutrosophic triplet.

(45, 57, 33) and (33, 57, 45) are neutrosophic triplets.

(35, 49, 35) is a neutrosophic triplet.

(77,49,77) is a neutrosophic triplet.

We are forced to think that there are elements x in Z_{84} for which there are neutral but has no anti x. Such study happens to be challenging and innovative.

It is a open problem to find which values will yield more number neutrosophic triplets.

Example 1.33. Let S = $\{Z_{51}, \times\}$ be the semigroup under product modulo 15.

$18 \times 18 \equiv 18 \pmod{51}$ and $34 \times 34 \equiv 34 \pmod{51}$ are the only idempotents

(3, 18, 6) and (6, 18, 6) are neutrosophic triplets.

(9, 18, 36) and (36, 18, 9) are neutrosophic triplets.

(27, 18, 12) and (12, 18, 27) are neutrosophic triplets.

(30, 18, 21) and (21, 18, 30) are neutrosophic triplets.

(39, 18, 24) and (24, 18, 39) are neutrosophic triplets.

(15, 18, 42) and (42, 18, 15) are neutrosophic triplets.

(45, 18, 48) and (48, 18, 45) are neutrosophic triplets.

(33, 18, 33) and (18, 18, 18) are neutrosophic triplets.

There are 16 neutrosophic triplets associated with 18.

34 does not induce any neutrosophic triplet.

Example 1.34. Let S = $\{Z_{34}, \times\}$ be the semigroup under product. The idempotents of S are $17 \times 17 \equiv 17 \pmod{34}$ and $18 \times 18 \equiv 18 \pmod{34}$.

(2, 18, 26) and (26, 18, 2) are neutrosophic triplets.

(4, 18, 30) and (30, 18, 4) are neutrosophic triplets.

(8, 18, 32) and (32, 18, 8) are neutrosophic triplets.

(16, 18, 16) and (18, 18, 18) are neutrosophic triplets.

(6, 18, 20) and (20, 18, 6) are neutrosophic triplets.

(12, 18, 10) and (10, 18, 12) are neutrosophic triplets.

(24, 18, 22) and (22, 18, 24) are neutrosophic triplets.

(14, 18, 28) and (28, 18, 14) are neutrosophic triplets.

There are exactly 16 neutrosophic triplets for $Z_{51.}$

Example 1.35. Let S = {Z_{95}, ×} be a semigroup under product modulo 85, 85 = 17 × 5.

51 × 51 ≡ 51 (mod 85) and 35 × 35 ≡ 35 (mod 85) are the idempotents of Z_{85}.

(5, 35, 75) and (75, 35, 5) are neutrosophic triplets.

(25, 35, 15) and (15, 35, 25) are neutrosophic triplets.

(40, 35, 20) and (20, 35, 40) are neutrosophic triplets.

(30, 35, 55) and (55, 35, 30) are neutrosophic triplets.

(65, 35, 45) and (45, 35, 65) are neutrosophic triplets.

(70, 35, 60), (60, 35, 70) and (35, 35, 35) are neutrosophic triplets of 35.

(10, 35, 80) and (80, 35, 10) are neutrosophic triplets.

(50, 35, 50) is a neutrosophic triplet.

(17, 51, 68) and (68, 51, 17) are neutrosophic triplets.

(34, 51, 34) and (51, 51, 51) are neutrosophic triplets.

There are 20 neutrosophic triplets together with (0, 0, 0) as its trivial neutrosophic triplet.

We see H = {(5, 35, 75), (75, 35, 5), (25, 35, 15), (15, 35, 25), (40, 35, 20), (20, 35, 40), (30, 35, 55), (55, 35, 30), (65, 35, 45), (45, 35, 65), (70, 35, 60), (60, 35, 70), (10, 35, 80), (80, 35, 10), (50, 35, 50) and (35, 35, 35)} are the neutrosophic triplets associated with the idempotent 35 which serves as the neutral elements of this collection.

Infact H under × is a cyclic group of order 16 with (35, 35, 35) as the identity element.

For (5, 35, 75) $\in$ H we have (5, 35, 75) × (5, 35, 75) = (25, 35, 15).

$(5, 35, 75)^3 = (40, 35, 20)$, $(5, 35, 75)^4 = (30, 35, 55)$

$(5. 35, 75)^5 = (65, 35, 45)$, $(5, 35, 75)^6 = (70, 35, 60)$

$(5, 35, 75)^7 = (10, 35, 80)$, $(5, 35, 75)^8 = (50, 35,50)$

$(5, 35,75)^9 = (80, 35, 10)$, $(5, 35,75)^{10} = (60, 35, 70)$

$(5, 35, 75)^{11} = (45, 35, 65)$, $(5, 35,75)^{12} = (55, 35, 30)$

$(5, 35, 75)^{13} = (20, 35, 40)$, $(5, 35, 75)^{14} = (15, 35,40)$ and

$(5, 35, 75)^{15} = (75, 35, 5)$ and $(5, 35,75)^{16} = (35, 35, 35)$.

Hence the claim.

K = {(17, 51, 68), (68, 51, 17), (51, 51, 51), (34, 51, 34)} is a cyclic group of order four.

$(17, 51, 68)^2 = (34, 51, 34)$, $(17, 51, 68)^3 = (68, 51, 17)$,

and $(17, 51, 68)^4 = (51, 51, 51)$.

We see $H \times K = \{(0, 0, 0)\}$.

$$P = \langle H \cup K \rangle = \{H \cup K \cup \{(0, 0, 0)\}\}.$$

Infact P is a semigroup of order 21 and it is not a monoid but P is a Smarandache semigroup.

Example 1.36. Let $S = \{Z_{55}, \times\}$ be the semigroup under product modulo 55.

The idempotents of Z_{55} are $11 \times 11 \equiv 11 \pmod{55}$ and $45 \times 45 \equiv 45 \pmod{55}$.

(5, 45, 20) and (20, 45, 5) are neutrosophic triplets.

(25, 45, 15) and (15, 45, 25) are neutrosophic triplets.

(10, 45, 10) and (45, 45, 45) are neutrosophic triplets.

(22, 11, 33) and (33, 11, 22) are neutrosophic triplets

(44, 11, 44) and (11, 11, 11) are neutrosophic triplets.

(30, 45, 40) and (40, 45, 30) are neutrosophic triplets.

(35, 45, 50) and (50, 45, 35) are neutrosophic triplets.

There are 14 neutrosophic triplets. We see H = {(5, 45, 20), (20, 45, 5), (25, 45, 15), (15, 45, 25), (30, 45, 40), (40, 45, 30), (35, 45, 50), (50, 45, 35), (10, 45, 10) and (45, 45, 45)} is a

neutrosophic triplet cyclic group with (45, 45, 45) as the identity $(35, 45, 50)^{10} = (45, 45\ 45)$.

K = {(22, 11, 33), (33, 11 22), (44, 11, 41), (11, 11, 11)} is a cyclic group of order four $(22, 11, 33)^4 = (11, 11, 11)$.

$H \times K = \{(0, 0, 0)\}$.

$\langle H \cup K \rangle = \{H \cup K \cup \{(0, 0, 0)\}\} = W$,

W is only a semigroup of order 15 and W is not a monoid but a Smarandache semigroup.

Example 1.37. Let $S = \{Z_{35}, \times\}$ be the semigroup under product modulo 15.

The idempotents of Z_{35} are $15 \times 15 \equiv 15 \pmod{35}$ and $21 \times 21 \equiv 21 \pmod{35}$.

(5, 15, 10) and (10, 15, 5) are neutrosophic triplets.

(25, 15, 30) and (30, 15, 25) are neutrosophic triplets.

(20, 15, 20) and (15, 15, 15) are neutrosophic triplets.

(7, 21, 28) and (28, 21, 7) are neutrosophic triplets.

(14, 21, 14) and (21, 21, 21) are neutrosophic triplets.

H = {(5, 15, 10), (10, 15, 15), (25, 25, 30), (30, 15, 25), (20, 15, 20), (15, 15, 15)} is a cyclic group of order six.

$(5, 15, 10)^6 = (15, 15, 15)$ as (15, 15, 15) acts as the identity element of H.

K = {(7, 21, 28), (28, 21, 7), (14, 21, 14), (21, 21, 21)} is again a cyclic group of order four with (21, 21, 21) as its identity. K is also a cyclic group of order four.

We see H × K = {(0, 0, 0)}.

$\langle H \cup K \rangle$ = {H ∪ K ∪ {(0, 0, 0)}} is only a semigroup not even a monoid of order 11.

Example 1.38. Let S = {Z_{143}, ×} be the semigroup under product modulo 143 = 11 × 13.

The idempotents of Z_{143} are 66 × 66 ≡ 66 (mod 143) and 78 × 78 ≡ 78 (mod 143).

(11, 66, 110) and (110, 66, 11), are neutrosophic triplets.

(121, 66, 88) and (88, 66, 121) are neutrosophic triplets.

(44, 66, 99) and (99, 66, 44) are neutrosophic triplets.

(55, 66, 22) and (22, 66, 55) are neutrosophic triplets.

(33, 66, 132) and (132, 66, 33) are neutrosophic triplets.

(77, 66, 77) and (66, 66, 66) are neutrosophic triplets.

(13, 78, 39) and (39, 78, 13) are neutrosophic triplets.

(26, 78, 91) and (91, 78, 26) are neutrosophic triplets.

(52, 78, 117) and (117, 78, 52) are neutrosophic triplets.

(104, 78, 130) and (130, 78, 104) are neutrosophic triplets.

(65, 78, 65) and (78, 78, 78) are neutrosophic triplets.

Infact we see there are two cyclic groups H and K which are of order 12 and 10 respectively where

H = {(11, 66, 110), (110, 66, 11), (121, 66, 88), (88, 66, 121), (44, 66, 99), (99, 66, 44), (55, 66, 22), (22, 66, 55), (33, 66, 132), (132, 66, 33), (77, 66, 77) and (66, 66, 66)} and

K = {(13, 78, 39), (39, 78, 13), (91, 78, 26), (26, 78, 91), (52,78, 117), (117, 78, 52), (104, 78, 130), (130, 78, 104), (65, 78, 65), (78, 78, 78)}.

Clearly $H \times K = \{(0, 0, 0)\}$ $\langle H \cup K \rangle = \{H \cup K \cup \{(0, 0, 0)\} = P$ is only a semigroup infact a Smarandache semigroup and not a monoid.

In view of all these we propose a conjecture as well as prove a theorem.

Theorem 1.1. *Let $S = \{Z_{pq}$, p and q are two distinct prime, $\times\}$ be the semigroup under $\times$ modulo pq.*

i) *S has only two idempotents given by mp and nq (m and $n \in Z_{pq} \setminus \{p, q\}$) such that they are neutrals.*

ii) *S has two cyclic groups H and K using the neutrals mp and nq of order $q - 1$ and $p - 1$ respectively.*

iii) $H \times K = \{(0, 0, 0)\}$.

iv) *$P = \{H \cup K \cup \{(0, 0, 0)\}\}$ is a only a semigroup which is not a monoid.*

v) *P is a S-semigroup.*

Proof is direct and hence left as an exercise to the reader.

Now we propose the conjecture.

Conjecture 1.11. Let S = {Z_{pq}, ×, p and q two distinct primes} be the semigroup under product modulo pq.

i) Prove Z_{pq} has only two idempotents.

ii) np and mq are two idempotents find m and n in terms of p or q.

Example 1.39. Let S = {Z_{105}, ×} be the semigroup under product modulo 105 = 3.5.7.

The idempotents of Z_{105} are 15 × 15 = 15 (mod 105),

21 × 21 = 21 (mod 105), 36 × 36 = 36 (mod 105)

70 × 70 ≡ 70 (mod 105), 85 × 85 = 85 (mod 105)

and 91 × 91 = 91 (mod 105).

(3, 36, 12) and (12, 36, 3) are neutrosophic triplets.

(9, 36, 39) and (39, 36, 9) are neutrosophic triplets.

(27, 36, 48) and (48, 36, 27) are neutrosophic triplets.

(81, 36, 51) and (51, 36, 81) are neutrosophic triplets.

(33, 36, 87) and (87, 36, 33) are neutrosophic triplets.

(99, 36, 99) and (36, 36, 36) are neutrosophic triplets.

(6, 36, 6) is a neutrosophic triplet.

(18, 36, 72) and (72, 36, 18) are neutrosophic triplets.

(54, 36, 24) and (24, 36, 54) are neutrosophic triplets.

(57, 36, 78) and (79, 36, 57) are neutrosophic triplets.

(66, 36, 96) and (96, 36, 66) are neutrosophic triplets.

(93, 36, 102) and (102, 36, 93) are neutrosophic triplets.

(69, 36, 69) is a neutrosophic triplet.

(30, 15, 60) and (60, 15, 30) are neutrosophic triplets.

(42, 21, 63) and (63, 21, 42) are neutrosophic triplets.

(84, 21, 84) and (21, 21, 21) are neutrosophic triplets.

(45, 15, 75) and (75, 15, 45) are neutrosophic triplets.

(90, 15, 90) and (15, 15, 15) are neutrosophic triplets.

(5, 85, 80) and (80, 85, 5) are neutrosophic triplets.

(25, 85, 100) and (100, 85, 25) are neutrosophic triplets.

(20, 85, 20) and (85, 85, 85) are neutrosophic triplets.

(10, 85, 40) and (40, 85, 10) are neutrosophic triplets.

(50, 85, 50) is a neutrosophic triplet.

(95, 85, 65) and (65, 85, 95) are neutrosophic triplets.

(55, 85, 55), (7, 91, 28) and (28, 91, 7) are neutrosophic triplets.

(49, 91, 49) and (91, 91, 91) are neutrosophic triplets

(14, 91, 14) is a neutrosophic triplet. (98, 91, 77) and (77, 91, 98) are neutrosophic triplets.

(56, 91, 56) is a neutrosophic triplet. (70, 70, 70), (35, 70, 50) and (50, 70, 35) are neutrosophic triplets.

There are six idempotents in Z_{105}, leading to six distinct cyclic groups of neutrosophic triplets given by

K_1 = {(50, 70, 35), (35, 70, 50), (70, 70, 70)}.

K_2 = {(7, 98, 25), (28, 91, 7), (49, 91, 49), (91, 91, 91), (14, 91, 14), (98, 91, 77), (77, 91, 98), (56, 91, 56)}.

K_3 = {(45, 15, 75), (75, 15, 45), (90, 15, 90), (15, 15, 15), (30, 15, 60), (60, 15, 30)}.

K_4 = {(55, 85, 55), (65, 85, 95), (95, 85, 65), (50, 85, 50), (10, 85, 40), (40, 85, 10), (20, 85, 20), (85, 85, 85), (25, 85, 100), (100, 85, 25), (5, 85, 80), (80, 85, 5)}.

K_5 = {(3, 36, 12), (12, 36, 3), (9, 36, 39), (39, 36, 9), (27, 36, 48), (48, 36, 27), (81, 36, 51), (51, 36, 81), (33, 36, 87), (87, 36, 33), (99, 36, 99), (36, 36, 36), (6, 36, 6), (18, 36, 72), (72, 36, 18), (54, 36, 24), (24, 36, 54), (57, 36, 78), (78, 36, 57), (66, 36, 99), (99, 36, 66), (93, 36, 102), (102, 36, 93), (69, 36, 69)} and

K_6 = {(42, 21, 63), (63, 21, 42), (84, 21, 84), (21, 21, 21)} are the groups of neutrosophic triplets.

$K_6 \times K_3 = \{(0, 0, 0)\}$, $K_3 \times K_5 = K_3$

$K_3 \times K_1 = \{(0, 0, 0)\}$, $K_3 \times K_4 = K_3$ and

$K_3 \times K_2 = \{(0, 0, 0)\}$.

$K_6 \times K_5 = K_6$.

$K_6 \times K_1 = \{(0, 0, 0)\}$, $K_6 \times K_2 = K_6$ and

$K_6 \times K_4 = \{(0, 0, 0)\}$.

$K_5 \times K_4 = K_3$.

$K_5 \times K_2 = K_6$ and $K_5 \times K_1 = \{(0, 0 ,0)\}$.

$K_4 \times K_2 = K_1$ $K_4 \times K_1 = K_3$ and $K_2 \times K_1 = K_{1.}$

These six groups behave only in the way described above.

Thus if P = $\langle K_1 \cup K_2 \cup K_3 \cup K_4 \cup K_5 \cup K_6\rangle \subseteq \{(K_1 \cup K_2 \cup K_3 \cup K_4 \cup K_5 \cup K_6\} \cup \{(0, 0, 0)\}\}$ and P will only be a semigroup and not a monoid.

However P will be a Smarandache semigroup.

If in Z_n, n is a product of three distinct odd primes then we see there are 6 groups of neutrosophic triplets such that

$K_1 \cap K_j = \{(0, 0, 0)\}$, if $i \neq j$, $1 \leq i, j \leq 6$, n = 105 = 3.5.7.

Example 1.40. Let S = $\{Z_{165}, \times\}$ be the semigroup under $\times$ modulo 165.

The idempotents of Z_{165} are

$55 \times 55 = 55 \pmod{165}$, $45 \times 45 \equiv 45 \pmod{165}$

$66 \times 66 = 66 \pmod{165}$, $100 \times 100 = 100 \pmod{165}$ and

$121 \times 121 = 121 \pmod{165}$, $111 \times 111 = 111 \pmod{165}$.

(3, 111, 147) and (147, 111, 3) are neutrosophic triplets.

(9, 111, 159) and (159, 111, 9) are neutrosophic triplets.

(27, 111, 108) and (108, 111, 27) are neutrosophic triplets.

(81, 111, 36) and (36, 111, 81) are neutrosophic triplets.

(78, 111, 12) and (12, 111, 78) are neutrosophic triplets.

(69, 111, 114) and (114, 111, 69) are neutrosophic triplets.

(42, 111, 93) and (93, 111, 42) are neutrosophic triplets.

(126, 111, 141) and (141, 111, 126) are neutrosophic triplets.

(48, 111, 102) and (102, 111, 48) are neutrosophic triplets.

(144, 111, 144) and (111, 111, 111) are neutrosophic triplets.

(6, 111, 156) and (156, 111, 6) are neutrosophic triplets.

(18, 111, 162) and (162, 111, 18) are neutrosophic triplets.

(54, 111, 54) is a neutrosophic triplet.

(5, 100, 20) and (20, 1000, 5) are neutrosophic triplets.

(25, 100, 70) and (70, 100, 25) are neutrosophic triplets.

(125, 100, 80) and (80, 100, 125) are neutrosophic triplets.

(130, 100, 115) and (115, 100, 130) are neutrosophic triplets.

(155, 100, 155) and (100, 100, 100) are neutrosophic triplets.

(10, 100, 10) is a neutrosophic triplet.

(50, 100, 35) and (35, 100, 50) are neutrosophic triplets.

(85, 100, 40) and (40, 100, 85) are neutrosophic triplets.

(95, 100, 140) and (140, 100, 95) are neutrosophic triplets.

(145, 100, 160) and (160, 100, 145) are neutrosophic triplets.

(65, 100, 65) is a neutrosophic triplet.

(15, 45, 69) and (69, 45, 45) are neutrosophic triplets.

(60, 45, 141) and (141, 45, 60) are neutrosophic triplets.

(75, 45, 159) and (159, 45, 75) are neutrosophic triplets.

(135, 45, 81) and (81, 45, 135) are neutrosophic triplets.

(22, 66, 33) and (33, 66, 22) are neutrosophic triplets.

(99, 66, 154) and (154,66, 99) are neutrosophic triplets.

(132, 66, 88) and (88, 66, 132) are neutrosophic triplets with 66 as the neutral element. (66, 66, 66) is a neutrosophic triplet, which acts as the identity for all elements with 66 as the neutrosophic element.

The reader is left with the task of finding whether the set

{(66, 66, 66), (88, 66, 132), (132, 66, 88), (154, 66, 99), (99, 66, 154), (22, 66, 33), (33, 66, 22)}

forms an abelian group with (66, 66, 66) as the multiplicative identity modulo 165.

Further the reader is left with the task of finding the largest neutrosophic triplet groups using Z_{165}.

It is conjectured that larger the number idempotents in Z_n the bigger is the neutrosophic triplets collection.

Conjecture 1.12. Let S = $\{Z_n, \times\}$ where n = 3m where is of the form pq where p and q are odd primes different from 3.

Can we say Z_n has only six idempotents?

We see in case of $n_1 = 105$ and $n_2 = 165$ where

$n_1 = 3 \times 5 \times 7$ and $n_2 = 3 \times 5 \times 11$ has only 6 idempotents.

Now we study the case when one of p and q is also 3 first by an example.

Example 1.41. Let $S = \{Z_{45}, \times\}$ be the semigroup under product modulo 45.

The idempotents of Z_{45} are

$10 \times 10 = 10 \pmod{45}$ and $36 \times 36 \equiv 36 \pmod{45}$.

Clearly we see if $n = 3^2 \times p$ where p is a prime then Z_n has only two idempotents as $45 = 3^2 \times 5$.

The neutrosophic triplets associated with the idempotents 10 and 36 are as follows.

(5, 10, 20) and (20, 10, 5) are neutrosophic triplets associated with the idempotent 10 of Z_{45}.

(25,10, 40) and (40, 10, 25) are also neutrosophic triplets associated with the idempotent 10.

(35, 10, 35) and (10, 10, 10) are neutrosophic triplets.

We first find the algebraic structure enjoyed by the set K = {(10, 10, 10), (35, 10, 35), (25, 10, 40), (40, 10, 25), (5, 10, 20) and (20, 10, 5)} under product modulo 45.

We construct the table of product in the following.

×	(10,10,10)	(35,10,35)	(25,10,40)	(40,10,25)	(5,10,20)	(20,10,5)
(10,10,10)	(10,10,10)	(35,10,35)	(25,10,40)	(40,10,25)	(5,10,20)	(20,10,5)
(35,10,35)	(35,10,35)	(10,10,10)	(20,10,5)	(5,10,20)	(40,10,25)	(25,10,40)
(25,10,40)	(25,10,40)	(20,10,5)	(40,10,25)	(10,10,10)	(35,10,35)	(5,10,20)
(40,10,25)	(40,10,25)	(5,10,20)	(10,10,10)	(25,10,40)	(20,10,5)	(35,10,35)
(5,10,20)	(5,10,20)	(40,10,25)	(35,10,35)	(20,10,5)	(25,10,40)	(10,10,10)
(20,10,5)	(20,10,5)	(25,10,40)	(5,10,20)	(35,10,35)	(10,10,10)	(40,10,25)

Clearly K is a group of order six and (10, 10, 10) acts as the identity element.

Now we find the neutrosophic triplets associated with 36.

J = {(36, 36, 36), (9, 36, 24), (24, 36, 9), (18, 36, 12), (12, 36, 18)}.

Infact we find it difficult to find for 3, 6, 15, 27, 30, 33, 39 and 42 anti elements however we have in some cases neutral elements, we shall define for those elements which has neutral elements but no anti element.

Such study is very different and difficult from usual neutrosophic triplet groups so we define only for those elements which has only neutrals but has no anti element so in case of these we define them to be duplets. They are called Neutrosophic Duplets and were introduced by Smarandache [23 - 25] in 2016.

We will illustrate them by some examples.

Example 1.42. Let S = $\{Z_{45}, \times\}$ be the semigroup under product modulo 45. We see the element 15 is a duplet with neutral element 10.

Interested reader can find whether such pair exist.

Study in this direction is innovative and interesting.

We define now duplet element of a semigroup.

Definition 1.4 *Let* $S = \{Z_n, \times\}$ *be the semigroup under* $\times$.

Let n_1 *and* n_2 *be any two neutral elements of* Z_n *if there is an element* $x \in Z_n$ *with* $xn_1 = n_1x = x$ *that is* n_i *is a neutral element of* Z_n *and if there does exist a* $y \in Z_n$ *with* $x \times y = y \times x = n_1$ *then we define* $\{x, n_1\}$ *to be the duplet.*

Interested reader can characterize such duplets.

Theorem 1.2. *Let* $S = \{Z_{2p}, \times\}$ *be the semigroup under product modulo 2p, p a prime. S does not contribute to duplets.*

Proof. Follows from the very fact that the neutral elements of Z_{2p} for any p a prime has neutrosophic triplet groups associated

with it. Hence Z_{2p} has no duplets. The elements p and p + 1 are the neutral elements of Z_{2p}.

Now we show if $n = 2^2 . 3$ or $n = 3^2 \times 2$ then that Z_n has non trivial duplets.

Example 1.43. Let S = $\{Z_{12}, \times\}$ be the semigroup under product modulo 12.

The neutral elements of Z_{12} are 4 and 9.

We give the elements which contribute to the neutrosophic triplet groups; (8, 4, 8), (4, 4, 4), (3, 9, 3) and (9, 9, 9).

(15, 9, 6) and (6, 9, 15) are neutrosophic triplet groups.

Example 1.44. Let S = $\{Z_{18}, \times\}$ be the semigrouop under product. 9 and 10 are the only idempotents (2, 10, 4) is a neutrosophic triplet. (4, 10, 16) is also a triplet. (8, 10, 8) and (10, 10, 10) are again neutrosophic triplets groups.

3 does not contribute to neutrosophic triplet group for $3 \times 10 \equiv 12 \pmod{18}$ and $3 \times 9 \equiv 9 \pmod{18}$.

However we define (3, 9, 3) as quasi neutrosophic triplet group. For 9 does not act as a neutral element of 3 but acts as a element which converts it to 9 and the anti of 3 is itself.

Now consider $6 \in Z_{18}$, (6, 10) is only a duplet for 6x = 10 for and x. (12, 10) is again a duplet.

(15, 9, 15) is again a quasi neutrosophic triplet as

$15 \times 9 \equiv 9 \pmod{18}$ and $15 \times 15 = 9 \pmod{18}$.

Thus the semigroup $\{Z_{18}, \times\}$ where $18 = 2 \times 3^2$ behaves in a unique way.

This has neutrosophic trplet groups given by the neutral element 10 which is as follows.

A = {(10, 10, 10), (8, 10,8), (2, 10,14), (14,10, 2), (4, 10, 16) and (16, 10, 4)} order of A is 8. This has two quasi neutrosophic triplets given by B = {(9, 9, 9), (3, 9, 3) and (15, 9, 15). Order of B is three.

This has duplets given by C = {(12,10) and (6, 10)}. We will first test whether A forms a group under component wise multiplication modulo 18.

We construct the table for A in the following.

×	(10, 10, 10)	(2, 10, 14)	(14, 10, 2)
(10, 10, 10)	(10, 10, 10)	(2, 10, 14)	(14, 10, 2)
(2, 10, 14)	(2, 10, 14)	(4, 10, 16)	(10, 10, 10)
(14, 10, 2)	(14,10, 2)	(10, 10, 10)	(16, 10, 4)
(4, 10, 16)	(4, 10, 16)	(8, 10, 8)	(2, 10, 4)
(16, 10, 4)	(16, 10, 4)	(14, 10, 2)	(8, 10, 8)
(8, 10, 8)	(8, 10, 8)	(16, 10, 4)	(4, 10, 16)

(4,10,16)	(16, 10, 4)	(8, 10, 8)
(4, 10, 16)	(16, 10, 4)	(8, 10, 8)
(8, 10, 8)	(14, 10, 2)	(16, 10, 4)
(2, 10, 14)	(8, 10, 8)	(4, 10, 16)
(16, 10, 4)	(10, 10, 10)	(14, 10, 2)
(10, 10, 10)	(4, 10, 16)	(2, 10, 14)
(14, 10, 2)	(2, 10, 14)	(10, 10, 10)

Clearly A is an abelian group under product modulo 18. We will first test whether A is a cyclic group and if so find the generator of A.

Consider (2, 10, 14) we first find

$(2, 10, 14) \times (2, 10, 14) \pmod{18}$

$= (4, 10, 16) = (2, 10, 14)^2$.

We find $(2, 10, 14)^3 = (4, 10, 16) \times (2, 10, 14) = (8, 10, 8)$.

Next we find $(2, 10, 14)^4 = (8, 10, 8) \times (2\ 10, 14) = (16, 10, 4)$.

Now $(2, 10, 14)^5 = (16, 10, 4) \times (2, 10, 14) = (14, 10, 2)$.

Finally $(2, 10, 14)^5 = (14, 10, 2) \times (2, 10, 14) = (10, 10, 10)$.

Thus (2, 10, 14) generates A, hence A is a cyclic group of order 6.

It is clearly seen that B and C are not even closed under product.

In view of all these we find the neutrosophic triplet groups, duplets and quasi neutrosophic triplets of $\{Z_{50} \times\}$.

Example 1.45. Let S = $\{Z_{50}, \times\}$ be the semigroup under product modulo 50. Clearly $50 = 2 \times 5^2$.

The idempotents of Z_{50} are 25 and 26.

Now we find the collection of neutrosophic triplet groups associated with the neutral element 26.

(2, 26, 38) and (38, 26, 2) are neutrosophic triplet groups.

(4, 26, 44) and (44, 26, 4) are neutrosophic triplet groups.

(8, 26, 22) and (22, 26, 8) are neutrosophic triplet groups.

(16, 26, 36) and (36, 26, 16) are neutrosophic triplet groups.

(32, 26, 18) and (18, 26, 32) are neutrosophic triplet groups.

(14, 26, 34) nd (34, 26, 14) are neutrosophic triplet groups.

(28, 26, 42) and (42, 26, 28) are neutrosophic triplet groups.

(6, 26, 46) and (46, 26, 6) are neutrosophic triplet groups.

(12, 26, 48) and (48, 26, 12) are neutrosophic triplet groups.

(24, 26, 24) and (26, 26, 26) are neutrosophic triplet groups.

(10, 26) is a duplet.

(15, 25, 15) is a quasi neutrosophic triplet group.

(20, 26) is a duplet.

(30, 26) is a duplet.

(40, 26) is a duplet.

(35, 25, 35) is a quasi neutrosophic triplet group.

(45, 25, 45) is a quasi neutrosophic triplet group.

(25, 25, 25) is a trivial neutrosophic triplet group or a quasi neutrosophic triplet group.

It is pertinent to record all non units of Z_{50} either form neutrosophic triplet groups or quasi neutrosophic triplet groups or duplet.

Now we test the probable algebraic structure enjoyed by these three groups neutrosophic triplet groups, quasi neutrosophic triplet groups and duplets.

Let X = {(26, 26, 26), (2, 26, 38), (38, 26, 2), (4, 26, 44), (44, 26, 4), (8, 26, 22), (22, 26, 8), (16, 26, 36), (36, 26, 16), (32, 26, 18), (18, 26, 32), (14, 26, 34), (34, 26, 14), (28, 26, 42), (42, 26, 28), (6, 26, 46), (46, 26, 6), (12, 26, 48), (48, 26, 12)

and (24, 26, 24)} be the collection of all neutrosophic triplet group.

Y = {(10, 26), (20, 26), (30, 26) and (40, 26)} be the collection of all neutrosophic duplets.

Z = {{15, 25, 15), (5, 25, 5), (25, 25, 25), (35, 25, 35) and (45, 25, 45)} be the collection of all quasi neutrosophic triplet groups.

We first find the table of Z under product modulo 50.

×	(15, 25, 15)	(25, 25, 25)
(15, 25, 25)	(25, 25, 25)	(25, 25, 25)
(25, 25, 25)	(25, 25, 25)	(25, 25, 25)
(35, 25, 35)	(25, 25, 25)	(25, 25, 25)
(45, 25, 45)	(25, 25, 25)	(25, 25, 25)
(5, 25, 5)	(25, 25, 25)	(25, 25, 25)

(35,25, 35	(45, 25, 45)	(5, 25, 5)
(25, 25, 25)	(25, 25, 25)	(25, 25, 25)
(25, 25, 25)	(25, 25, 25)	(25, 25, 25)
(25, 25, 25)	(25, 25, 25)	(25, 25, 25)
(25, 25, 25)	(25, 25, 25)	(25, 25, 25)
(25, 25, 25)	(25,25, 25)	(25, 25, 25)

This Z forms a unique type of semigroup where all products lead (25, 25, 25).

Infact Z is a unique type of monoid under product modulo 50.

Now we find the algebraic structure enjoyed by Y. The table for Y ∪ (0, 26) is as follows.

×	(10, 26)	(20, 26)	(30,26)	(40,26)	(0, 26)
(10, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)
(20, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)
(30, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)
(40, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)
(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)	(0, 26)

We see Y ∪ {(0, 26)} the collection of all duplets under product modulo 50 is again a unique type of monoid which results in (0, 26). Order of Y ∪ {(0, 26)} is 5.

However if the trivial duplet pair (0, 26) is not added the set Y will suffer under non closure property under product modulo 50.

However Z the set of all quasi neutrosophic triple groups forms a unique type of semigroup under product modulo 50.

Both Y and Z are of order 56.

Now we find the structure enjoyed by X under product modulo 50. Clearly order of X is 20.

Let us consider (2, 26, 38) ∈ X.

$(2, 26, 38)^2 = (2, 26, 38) \times (2, 26, 38) = (4, 26, 44)$,

$(2, 26, 38)^3 = (4, 26, 44) \times (2, 26, 38) = (8, 26, 22)$,

$(2, 26, 38)^4 = (8, 26, 22) \times (2,26, 38) = (16, 26, 36)$,

$(2, 26, 38)^5 = (16, 26, 36) \times (2, 26, 38) = (32, 26, 18)$,

$(2, 26, 38)^6 = (2, 26,38) \times (32, 26, 18) = (14, 26, 34)$,

$(2, 26, 38)^7 = (14, 26, 34) \times (2, 26, 38) = (28, 26, 42)$,

$(2, 26, 38)^8 = (28, 26, 42) \times (2, 26, 38) = (6, 26, 46)$,

$(2, 26, 38)^9 = (6, 26, 46) \times (2, 26, 38) = (12, 26, 48)$,

$(2, 26, 38)^{10} = (24, 26, 24) = (2, 26, 38) \times (12, 26, 48)$,

$(2, 26, 38)^{11} = (48, 26, 12) = (2, 26, 38) \times (24, 26, 24)$,

$(2, 26, 38)^{12} = (48, 26, 12) \times (2, 26, 38) = (46, 26, 6)$,

$(2, 26, 38)^{13} = (46, 26, 6) \times (2, 26, 38) = (42, 26, 28)$,

$(2, 26, 38)^{14} = (42, 26, 28) \times (2, 26, 38) = (34, 26, 14)$,

$(2, 26, 38)^{15} = (34, 26,14) \times (2, 26, 38) = (18, 26,32)$,

$(2, 26, 38)^{16} = (18, 26, 32) \times (2, 26, 38) = (36, 26, 16)$,

$(2, 26, 38)^{17} = (36, 26, 16) \times (2, 26, 38) = (22, 26, 8)$,

$(2, 26, 38)^{18} = (22, 26, 8) \times (2, 26, 38) = (44, 26, 4)$,

$(2, 26, 38)^{19} = (38, 26, 2) = (2, 26, 38) \times (44, 26, 4)$,

$(2, 26, 38)^{20} = (38, 26, 2) \times (2, 26, 38) = (26, 26, 26)$.

Thus X is proved to be a cyclic group of order 20 under product modulo 50.

In view of all these we first propose only one conjecture.

Conjecture 1.13. Let $S = \{Z_n, \times\}$ where $n = 2 \times p^2$, p an odd prime.

i) The collection of all neutrosophic triplet groups forms a cyclic group of even order under product modulo $n = 2p^2$.

ii) The collection of all duplets forms a special type of monoid of order p under product modulo n.

iii) The collection of all quasi neutrosophic triplet groups forms a unique type of semigroup of order p under product modulo. $2p^2$.

Before we make one more conjecture we propose another example.

Example 1.46. Let $S = \{Z_{98}, \times\}$ be the semigroup under product modulo 98.

The neutral elements of Z_{98} are 49 and 50.

The neutrosophic triplet groups associated with 50 are (2, 50, 74) and (74, 50, 2) are neutrosophic triplet groups.

(4, 50, 86) and (86, 50, 4) are neutrosophic triplet groups.

(8, 50, 92) and (92, 50, 8) are neutrosophic triplet groups.

(16, 50, 46) and (46, 50,16) are neutrosophic triplet groups.

(32, 50, 72) and (72, 50, 32) are neutrosophic triplet groups.

(64, 50, 36) and (36, 50, 64) are neutrosophic triplet groups.

(30, 50, 18) and (18, 50, 30) are neutrosophic triplet groups.

(60, 50, 58) and (58, 50, 60) are neutrosophic triplet groups.

(22, 50, 78) and (78, 50, 22) are neutrosophic triplet groups.

(44, 50, 88) and (88, 50, 44) are neutrosophic triplet groups.

(6, 50, 90) and (90, 50, 6) are neutrosophic triplet groups.

(12, 50, 94) and (94, 50, 12) are neutrosophic triplet groups.

(24, 50, 96) and (96, 50, 24) are neutrosophic triplet groups.

(48, 50, 48) and (50, 50, 50) are neutrosophic triplet groups.

(10, 50, 54) and (54, 50, 10) are neutrosophic triplet groups.

(20, 50, 76) and (76, 50, 20) are neutrosophic triplet groups.

(40, 50, 38) and (38, 50, 40) are neutrosophic triplet groups.

(80, 50, 68) and (68,50, 80) are neutrosophic triplet groups.

(62, 50, 34) and (34,50, 62) are neutrosophic triplet groups.

(26, 50, 66) and (66, 50, 26) are neutrosophic triplet groups.

(52, 50, 82) and (82, 50, 52) are neutrosophic triplet groups.

We see if A = {(2, 50, 74), (74, 50, 2), (4, 50, 86), (86, 50, 4), (8, 50, 92), (92, 50, 8), (16, 50, 46), (46, 50, 16), (32, 50, 72), (72, 50, 32), (64, 50, 36), (36, 50, 64), (30, 50, 18), (18, 50, 30), (60, 50, 58), (58, 50, 60), (22, 50, 78), (78, 50, 22), (44, 50, 88), (88, 50,44), (6, 50, 90), (90, 50, 6), (12, 50, 94), (94, 50, 12), (24,50, 96), (96, 50, 24), (48, 50, 48), (50, 50, 50), (10, 50, 54), (54, 50, 10), (20, 50, 76), (76, 50,20), (40, 50, 38), (38, 50, 40), (80, 50, 68), (68, 50, 80), (62, 50,34), (34,50,62) (26, 50, 66), (66, 50, 26), (52, 50, 82) and (82, 50,52)} forms a group under product modulo 98 of order 42.

Now we find the collection of duplets.

B = {(7, 49, 7), (21, 4, 9, 21), (35, 4, 9, 35), (63, 49, 63), (77, 49, 77), (91, 49, 91) and (49, 49, 49)} are quasi neutrosophic triplet groups associated with the trivial neutral element 49.

Clearly o(B) = 7

{(14, 50), (28, 50), (56, 50), (70, 50), (42, 50), (84, 50) and (0, 50)} = C is the collection of all duplets which forms a unique type of semigroup under product modulo 98.

Similarly B the collection of all quasi neutrosophic triplet groups under product modulo 98 forms a unique type of semigroup of order 7.

In view of all these we propose yet some open conjectures.

Conjecture 1.14. Let S = $\{Z_n, \times\}$ be the semigroup under product modulo n, where n = $2p^2$ where p is any odd prime with only two neutral elements (or idempotents), p^2 and p^2+1.

i) The neutrosophic triplet groups collection A associated with the neutral element (or idempotent) $p^2 + 1$ is a cyclic group of order (p – 1) p.

ii) The set of all quasi neutrosophic triplet groups associated with p^2 is a special type of semigroup under product modulo $2p^2$ is of order p.

iii) The collection of duplets C associated with the idempotent $p^2 + 1$ is again a

special type of semigroup in which for every x, y $\in$ C. $x \times y = (0, p^2 + 1)$, product operation under modulo $2p^2$ $(x \neq y)$ and $x^2 = (0, p^2 + 1)$.

It is further pertinent to record that p^2 is only a trivial neutral element which can yield non trivial quasi neutrosophic triplet groups. However $p^2 + 1$ cannot yield nontrivial quasi neutrosophic triplet groups.

Further $p^2 + 1$ alone has the capacity to yield both neutrosophic triplet groups as well as duplets.

Next we study $\{Z_n, \times\}$ where $n = pq^2$ where both p and q two distinct odd primes for neutrosophic triplet groups, quasi neutrosophic triplet groups and duplet pairs.

Example 1.47 Let $S = \{Z_{45}, \times\}$ be a semigroup under product modulo 45. $45 = 3^2 \times 5$.

The neutral elements or idempotents of S are 10 and 36 (5, 10, 20) and (20, 10, 5) are neutrosophic triplet groups of the neutral element 10.

(25, 10, 40) and (40, 10, 25) are neutrosophic triplet groups,

(35, 10, 35) and (10, 10, 10) are neutrosophic triplet groups.

(6, 36, 6) is a neutrosophic triplet group.

(9, 36, 9) is a neutrosophic triplet group.

(15, 10) is a duplet, (3, 10) is a duplet, (21, 36, 21) is a quasi neutrosophic triplet group.

(27, 36, 18) and (18, 36, 27) are neutrosophic triplet groups.

Now we see 3, 12, 33 and 39 which are only multiplies of three though is not a unit in Z_{45} do not contribute to any neutrosophic triplet group or duplet or quasi neutrosophic triplet groups.

However 21 which is a multiple of 3 with 7 is a quasi neutrosophic triplet group.

Further let A = {(5, 10, 20), (20, 10, 5), (25, 10, 40), (40, 10, 25), (35, 10, 35), (10, 10, 10)} be the collection of all neutrosophic triplet group associated with the neutral element 10.

B = {(9, 36, 9), (36, 36, 36), (27, 36, 18), (18, 36, 27)} be the collection of all neutrosophic triplet groups associated with the neutral element 36.

{(6, 36, 6), (21, 36, 21), (36, 36, 36)} be the collection of all quasi neutrosophic triplet groups associated with the neutral element 36.

{(15, 10), (30, 10), (0, 10)} be the collection of duplets associated with the neutral element 10.

We see when $n = p^2q$ where p and q are two distinct odd primes we are not in a situation to describe the set of neutrosophic triplet groups or duplets or quasi neutrosophic triplet groups.

Further even the neutral elements of 45 are 10 and 36 which seems to be very different from $2 \times p^2$, p an odd prime.

However in case of $45 = 3^2 \times 5$ we see $10 = 5 \times (3 - 1)$ and $36 = 3^2 \times (5 - 1)$. So can we say if $n = p^2q$, $p < q$; then $q \times (p - 1)$ and $p^2 \times (q - 1)$ are the two neutral elements of Z_n.

We now describe another example.

Example 1.48. Let $S = \{Z_n, \times\}$ where $n = 3^2 \times 7$ be the semigroup under product modulo n = 63. The idempotents or neutrals of Z_{63} are 28 and 36.

We now find the neutrosophic triplet groups, duplets and quasi neutrosophic triplet groups.

(7, 28, 49) and (49, 28, 7) are neutrosophic triplet groups.

(14, 28, 56) and (56, 28, 14) are neutrosophic triplet groups.

(35, 28, 35) and (28, 28, 28) are neutrosophic triplet groups.

(9, 36, 18) and (18, 36, 9) are neutrosophic triplet groups.

(27, 36, 27) and (36, 36, 36) are neutrosophic triplet groups.

(45, 36, 54) and (54 36, 45) are neutrosophic triplet groups.

One can calculate the duplets and quasi neutrosophic triplet groups.

However finding them happens to be a difficult task.

The neutrosophic triplet groups associated with Z_{63} is a difficult task.

Further it is pertinent to record that 36 and 28 contribute non trivial neutrosophic triplet groups.

Here let Y ={(7, 28, 49), (49, 28, 7), (14,28, 56), (56, 28, 14), (28, 28, 28), (35, 28, 35)} and

Z = {(9, 36, 18), (18, 36, 9), (27, 36, 27), (36, 36, 36), (45, 36, 54), (54, 36, 45)} be the nontrivial neutrosophic triplet groups collection associated with the neutral elements 28 and 36 respectively.

We will first find the tables of Y and Z.

Table of Y is as follows:

×	(7, 28, 49)	(49, 28, 7)	(14, 28, 56)
(7, 28, 49)	(49,28,7)	(28, 28, 28)	(35, 28, 35)
(49, 28, 7)	(28, 28, 28)	(7, 28, 49)	(56, 28, 14)
(14, 28, 56)	(35, 28, 35)	(56, 28, 14)	(7, 28, 49)
(56, 28, 14)	(14, 28, 56)	(35, 28, 35)	(28, 28, 28)
(28, 28, 28)	(7, 28, 49)	(49, 28, 7)	(14, 28, 56)
(35, 28, 35)	(56, 28, 14)	(14, 28, 56)	(49, 28, 7)

(56, 28, 14)	(28, 28, 28)	(35, 28, 35)
(14, 28, 56)	(7, 28, 49)	(56, 28, 14)
(35, 28, 35)	(49, 28, 7)	(14, 28, 56)
(28, 28, 28)	(14, 28, 56)	(49, 28, 7)
(49, 28, 7)	(56, 28, 14)	(7, 28, 49)
(56, 28, 14)	(28, 28, 28)	(35, 28, 35)
(7, 28, 49)	(35, 28, 35)	(28, 28, 28)

Clearly Y is a group of order six. We have to check whether Y is cyclic or not.

Consider (7, 28, 49) we see

$(7, 28, 49) \times (7, 28, 49) = (49, 28, 7)$ and

$(49, 28, 7) \times (7, 28, 49) = (28, 28, 28)$.

Thus $(7, 28, 49)^3 = (28, 28, 28)$ the identity.

Now if we take (14, 28, 56) then $(14, 28, 56)^2 = (7, 28, 49)$

$(14, 28, 56)^3 = (7, 28, 49) \times (14, 28, 56) = (35, 28, 35)$

$(14, 28, 56)^4 = (35, 28, 35) \times (14, 28, 56) = (49, 28, 7)$

$(14, 28, 56)^5 = (49, 28, 7) \times (14,28, 56) = (56, 28, 14)$

Finally $(14, 28, 56)^6 = (56, 28, 14) \times (14, 28, 56) = (28, 28,28)$.

Thus (14, 28, 56) is the generator of the cyclic group Y with (28, 28, 28) as the identity.

Interested reader can find if any other element generates Y.

Now we give the table for Z in the following.

×	(9, 36, 18)	(18, 36,9)	(27, 36,27)	(36, 36, 36)	(45, 36,54)	(54, 36, 45)
(9, 36, 18)	(18, 36, 9)	(36, 36, 36)	(54, 36, 45)	(9, 36, 18)	(27, 36, 27)	(45, 36, 54)
(18, 36, 9)	(36, 36, 36)	(9, 36, 18)	(45, 36, 54)	(18, 36, 9)	(54, 36, 45)	(27, 36, 27)
(27, 36, 27)	(54, 36,45)	(45, 36, 54)	(36, 36, 36)	(27, 36, 27)	(18, 36, 9)	(9, 36, 18)
(36, 36, 36)	(9, 36,18)	(18, 36, 9)	(27, 36, 27)	(36, 36, 36)	(45, 36, 54)	(54, 36, 45)
(45, 36, 54)	(27, 36, 27)	(54, 36, 45)	(18, 36, 9)	(45, 36, 54)	(9, 36, 18)	(36, 36, 36)
(54, 36, 45)	(45, 36, 54)	(27, 36, 27)	(9, 36, 18)	(54, 36, 45)	(36, 36, 36)	(18, 36, 9)

Thus Z is also a group with (36, 36, 36) as its multiplicative identity.

We now find out whether Z is cyclic or not under the product operation modulo 63.

Consider $(9, 36, 18) \in Z$; $(9, 36, 18)^2 = (18, 36, 9)$;

$(9, 36, 18)^3 = (18, 36, 9) \times (9, 36, 18) = (36, 36, 36)$.

Thus the order of (9, 36, 18) is only three and does not generate Z.

Consider $(45, 36, 54) \in Z$; $(45, 36, 54)^2 = (9, 36, 18)$,

$(45, 36, 54)^3 = (9, 36, 18) \times (45, 36, 54) = (27, 36, 27)$,

$(45, 36, 54)^4 = (45, 36, 54) \times (27, 36, 27) = (18, 36, 9)$,

$(45, 36, 54)^5 = (18, 36, 9) \times (45, 36, 54) = (54, 36, 45)$

and $(45, 36, 54)^6 = (54, 36, 45) \times (45, 36, 54) = (36, 36, 36)$.

Thus as order of (45, 36, 54) is six (45, 36, 54) generates the Z so Z is a cyclic group of order six with (36, 36, 36) as identity.

Now we test for quasi neutrosophic triplet groups and duplets in Z_{63} under product modulo 63.

(15, 36, 15) is a quasi neutrosophic triplet group.

(57, 36), (42, 28) and (21, 28) are some of the duplets.

The reader is left with the task of finding both duplets associated with 28 and 36 as well as quasi neutrosophic triplet groups associated with 28 and 36.

We give one more example before we give any of the related observations.

Example 1.49. Let S = $\{Z_{99}, \times\}$ be the semigroup under product modulo 99. Clearly $99 = 3^2 \times 11$ so 99 is of the form 3^2p where p is an odd prime.

First we find the related neutral elements of Z_{99}.

45 and 55 are the only neutral elements or idempotents of Z_{99}.

We now find the elements which contribute to neutrosophic triplet groups by 55.

(11, 55, 77) and (77, 55, 11) are neutrosophic triplet group associated with the neutral element 55.

(22, 55, 88) and (88, 55, 22) are neutrosophic triplet groups.

(44, 55, 44) and (55, 55, 55) are neutrosophic triplet groups.

(12, 45, 12) and (45, 45, 45) are neutrosophic triplet groups.

(18, 45,63) and (63, 45, 18) are neutrosophic triplet groups.

(27, 45, 9) and (9, 45, 27) are neutrosophic triplet groups.

(90, 45, 72) and (72, 45, 90) are neutrosophic triplet groups.

(36, 45, 81) and (81, 45, 36) are neutrosophic triplet groups.

(54, 45, 54) is a neutrosophic triplet group.

(33, 55, 33) is a neutrosophic triplet group. The author is left with the task of finding the group of neutrosophic triplet groups associated with 45 and 55.

We find some of the neutrosophic triplet groups associated with 55.

{(11, 55, 77), (77, 55, 11), (55, 55, 55), (44, 55, 44), (22, 55, 88) (88, 55, 22)}.

The reader is left with task of finding quasi neutrosophic triplet groups and duplets.

We find it a challenging task to the collection of all duplet pairs, quasi neutrosophic triplet groups and neutrosophic triplet groups and study the algebraic structure enjoyed by them.

Finally we give an example where n = 48 = $2^4 3$.

Example 1.50. Let S = $\{Z_{48}, \times\}$ be the semigroup under product modulo 48. The neutral elements (idempotents) of Z_{48} are 16 and 33.

The neutrosophic triplet groups associated with the neutral element 16 are ({16, 16, 16), (32, 16, 32)}.

We see A is only a cyclic group of order two.

However for the neutral element 33 we give some of the neutrosophic triplet groups;

{(3, 33, 27), (27, 33, 3), (9, 33, 9), (15, 33, 15), (33, 33, 33), (21, 33, 21)}.

However we are forced to make the following open conjectures.

Can the collection of neutrosophic triplets associated with Z_n, when $n = 2^t p$, $t \geq 2$ and p an odd prime for any of the neutral elements be groups of order greater than 2 when the neutral element is an even number?

Study the above question analogously for any Z_n, $n = p^m q$, $m \geq 2$ p and p two distinct odd primes.

We suggest the following problems for the reader.

Problems

1. Obtain all special features associated with the collection of neutrosophic triplet groups.

2. When will the number of neutral elements set be large?

3. Can $n = 2p$ for Z_n, p an odd prime have more than two nontrivial neutral elements? Justify!

4. Find all the neutral elements of $S = \{Z_{422}, \times\}$.

5. Find all the neutral elements of $P = \{Z_{210}, \times\}$.

6. Which of the semigroups S or P has more number of neutral elements?

7. Can one prove the number of neutral elements depends not on the largeness of n in Z_n but the number of prime factors that can divide n?

8. Find all the neutral elements of Z_{45}.

9. Can Z_{101} have neutral elements?

10. Find all the neutrosophic triplet groups associated with 54 of S = $\{Z_{106}, \times\}$.

    a) Prove the collection of all neutrosophic triplet groups associated with the neutral element 54 $\in Z_{106}$ is a group G.

    b) Prove the group G is a cyclic group.

    c) Find the order of G.

11. Show S = $[Z_{46}, \times\}$ has pseudo primitive elements associated with set P = {2, 4, 6, 8, 10, …, 42, 44}.

12. Prove in $\{Z_n, \times\}$ where n = 3p, p an odd prime different from 3 has only two neutral elements. Generalize them and find its form.

13. Let S = $\{Z_{45}, \times\}$ be the semigroup under product modulo 15.

    i) Prove 10 and 6 are the only neutral elements of Z_{15}.

    ii) Find the neutrosophic triplet groups associated with 10 and 6.

    iii) Does the collection of neutrosophic triplet groups associated with the neutral element 10 form a group?

    iv) Find the order of the group in (iii) .

v) Can S have duplets?

vi) Does any of the neutral elements of S contribute to quasi neutrosophic triplet groups?

14. Let S = $\{Z_{560}, \times\}$ be the semigroup under product modulo 560.

a) Find all the neutral elements associated with Z_{560}.

b) Find the neutrosophic triplet groups collection associated with each of the neutral elements of Z_{560}.

c) Does these collections associated with the neutral element form a group or a semigroup?

d) Can we say if G and H are two neutrosophic triplet groups collection associated with two distinct neutral elements then $G \times H = \{(0, 0, 0)\}$?

e) Obtain any other special feature associated with S.

15. Let B = $\{Z_{510}, \times\}$ be the semigroup under product modulo 510.

i) Study questions (a) to (e) of problem (14) for this B.

ii) Can we say neutral elements of B contribute more number of neutrosophic triplet groups than that of S in problem (13)?

16. Prove if S = $\{Z_n, \times\}$ be a semigroup with n = $p_1 p_2 \ldots p_t$, p_i are distinct primes $t \geq 3$, $1 \leq i \leq t$, then Z_n has larger number of neutral elements than Z_m where $m = p^s q > n$ (p and q two distinct primes).

17. Find for the fixed t in problem (16) the total number of neutral elements.

18. Let S = $\{Z_{900}, \times\}$ be the semigroup under product modulo 900.

    i) Find all neutral elements of S.

    ii) Can neutral elements of S contribute duplets?

    iii) Does neutral elements of S contribute to quasi neutrosophic triplet groups?

    iv) Find all the neutrosophic triplet groups associated with each of the neutral elements.

    v) Study the algebraic structure enjoyed by each of the collection of neutrosophic triplet groups with every distinct nontrivial neutral element of S.

19. Let P = $\{Z_{418}, \times\}$ be the semigroup under product modulo 418.

    i) Study questions (i) to (v) of problem (18) for this P.

    ii) Which has more number of neutral elements S or P?

20. Characterize those numbers n of Z_n (n a non prime composite number) which has duplets.

21. Characterize those integers n for which Z_n contributes neutral elements which contributes quasi neutrosophic triplet groups.

22. Is it always true for a fixed Z_n if n has neutrals which contribute to duplets than it has other neutrals which contribute to quasi neutrosophic triplet groups?

23. Prove if S = $\{Z_n, \times\}$, n = pq, p and q two distinct primes than no neutral element in Z_n can contribute to duplets or quasi neutrosophic triplet groups.

24. Can we say for M = $\{Z_n, \times\}$, n = $p^t q$, p and q two distinct primes there are neutrals in Z_n which contribute to both duplets and quasi neutrosophic triplet groups?

25. Can we say for some n of Z_n we can have neutrals which contribute only to duplets and no neutrals exists in that Z_n which can contribute to quasi neutrosophic triplet groups or neutrosophic triplet groups?

26. Can we prove there exists n such that Z_n can have neutrals which contribute only to quasi neutrosophic triplet groups and not to duplets or neutrosophic triplet groups?

27. Is it mandatory if a neutral in Z_n contributes to quasi neutrosophic triplet groups then n is a product of several distinct powers of primes?

28. Prove the converse of problem given in (27) is in general not true.

29. Prove in $\{Z_n, \times\}$, the semigroup under product there are neutrals which contribute to duplets then the collection is a unique of semigroup in which product of every pair is $\{(0, t)\}$, where t is that special neutral element of Z_n which has contributed duplets.

30. Let $S = \{Z_n, \times\}$ be the semigroup under product modulo n. If $s \in Z_n$ is a neutral which contributes to a collection of quasi neutrosophic triplet groups them prove that collection is a special type of semigroup such that every pair of elements product in it is (s, s, s).

31. Can there exist a Z_n in which every neutral contributes only for duplets? (that is all neutrals in Z_n has the capacity only to create neutrosophic pairs)?

32. Does there exist a Z_n is which all neutrals contribute only to quasi neutrosophic triplet groups?

33. Let $S = \{Z_{648}, \times\}$ be the semigroup under product modulo 648.

    i) Find all neutrals of S.

ii) Can these neutrals contribute to nontrivial neutrosophic triplet group collection which forms a cyclic group?

iii) Can the neutrals of Z_{648} contribute to duplets alone, which forms a special type of semigroup?

iv) Can the neutral of Z_{648} contribute only to quasi neutrosophic triplet groups? Or is it mandatory a neutral which contribute to quasi neutrosophic triplet groups necessarily contributes to neutrosophic triplet groups also.

v) Find all neutrals of Z_{648} which contribute only to one type of neutrosophic triplet groups (does such neutrals exist in Z_{648}).

34. Let $S_1 = \{Z_{392}, \times\}$ be the semigroup under product modulo 392.

    i) Study questions (i) to (v) of problem (33) for this S_1.

    ii) Compare the properties enjoyed by S of problem in (33) with this S_1

35. Construct a semigroup S such that $S \times S = \{\text{identity}\}$ and S is of finite order.

36. Let $S = \{Z_{2p}, \times\}$, p an odd prime. $B = \{2Z_{2p} \setminus \{0\}\} \subseteq S$. Prove B has atleast one pseudo primitive element.

37. Can B in problem 36 have more than one pseudo primitive element?

38. Can $S = \{Z_{pq}, \times\}$, p and q different odd primes with $D = \{pZ_{pq} \setminus \{0\}\} \subseteq S$ have pseudo primitive elements?

39. Let $S = \{Z_{1219}, \times\}$, be the semigroup under product modulo 1219.

    i) $P_1 = \{53Z_{1219} \setminus \{0\}\} \subseteq S$; can P_1 have pseudo primitive elements?

    ii) Let $P_2 = \{23Z_{1219} \setminus \{0\}\} \subseteq S$; can P_2 have pseudo primitive elements?

Chapter Two

ALGEBRAIC STRUCTURES ON NEUTROSOPHIC TRIPLET GROUPS

In this chapter we study the algebraic structure enjoyed by the neutrosophic triplet groups built using Z_{pq}, p and q are prime numbers. In the earlier chapter we have studied the properties of neutrosophic triplets built using Z_n.

We see these neutrosophic triplet groups of Z_{2n} behave and enjoy properties different from Z_n when n is not of the above said form.

We will first illustrate this situation by examples.

Example 2.1. Let S = $\{Z_{22}, \times\}$ be the semigroup under product modulo 22. The idempotents of S are 11 and 12.

The elements that contribute for neutrosophic triplets from Z_{22} are A = {2, 4, 6, 8, 10, 14, 15, 18, 20, 12 and 11}.

We now get the neutrosophic triplets of Z_{22}.

Only idempotents can serve as neutral elements from the set A.

Infact the idempotent 11 cannot contribute to any nontrivial neutrosophic triplet and infact (11, 11, 11) is the trivial neutrosophic triplet

(2, 12, 6) and (6, 12, 2) are neutrosophic triplets associated with the idempotent 12.

(4, 12, 14) and (14, 12, 4) are neutrosophic triplets.

(8, 12, 18) and (18, 12, 8) are neutrosophic triplets.

(16, 12, 20) and (20, 12, 16) are neutrosophic triplets.

(10, 12, 10) and (12, 12, 12) are neutrosophic triplets.

Thus there are 10 nontrivial neutrosophic triplet groups all of which have only 12 to be the neutral element.

11 is the trivial neutral element of Z_{22}.

The set B = {(2, 12, 6), (6, 12, 2), (4, 12, 14), (14, 12, 4), (8, 12, 18), (18, 12, 8), (16, 12, 20), (20, 12, 6), (10, 12, 10), (12, 12, 12)} forms a group under modulo 22. $\{B, \times\}$ is defined as the neutrosophic triplet group - group.

In fact B is a cyclic group of order 10 and it is generated by (2, 12, 6).

For $(2, 12, 6)^{10}$ = (12, 12, 12) and (12, 12, 12) acts as the identity of B. (0, 0, 0) and (11, 11, 11) are only trivial neutrosophic triplets.

We see (11, 11, 11) × (11, 11, 11) = (11, 11, 11) and (0, 0, 0) × (11, 11, 11) = (0, 0, 0).

Finally (11, 11, 11) × x = (0, 0, 0) for all x ∈ B.

Further ⟨B ∪ {(0, 0, 0), (11, 11, 11)}⟩ = P is a semigroup which is not a monoid. In fact {(0, 0, 0), (11, 11, 11)} × B = {(0 0 0)}. Thus B is annulled by the element (11, 11, 11).

Example 2.2. Let W = {Z_{18}, ×} be the semigroup under product modulo 18. 9 × 9 ≡ 9 (mod 18), 10 × 10 = 10 (mod 18), 2 × 10 = 2 (mod 18);

(2, 10, 14) and (14, 10, 2) are neutrosophic triplets.

(4, 10, 16) and (16, 10, 4) are neutrosophic triplets.

(8, 10, 8) and (10, 10, 10) are neutrosophic triplets.

We see the neutrosophic triplets associated with the neutral element 10 are K = {(10, 10, 10), (2, 10, 14), (14, 10, 2), (16, 10, 4), (4, 10, 16), (8, 10, 8)}. The table for K is ;

×	(10, 10, 10)	(2, 10, 14)	(14, 10, 2)
(10, 10, 10)	(10, 10, 10)	(2, 10, 14)	(14, 10, 2)
(2, 10, 14)	(2, 10, 14)	(4, 10, 16)	(10, 10, 10)
(14, 10, 2)	(14, 10, 2)	(10, 10, 10)	(16, 10, 4)
(8, 10, 8)	(8, 10, 8)	(16, 10, 4)	(4, 10, 16)
(16, 10, 4)	(16, 10, 4)	(14, 10, 2)	(8, 10, 8)
(4, 10, 16)	(4, 10, 16)	(8, 10, 8)	(4, 10, 16)

(8, 10, 8)	(16, 10, 4)	(4, 10, 16)
(8, 10, 8)	(16, 10, 4)	(4, 10, 16)
(16, 10, 4)	(14, 10, 2)	(8, 10, 8)
(4, 10, 16)	(8, 10, 8)	(4, 10, 16)
(10, 10, 10)	(2, 10, 14)	(14, 10, 2)
(2, 10, 14)	(4, 10, 16)	(10, 10, 10)
(14, 10, 2)	(10, 10, 10)	(16, 10, 4)

In fact this is an abelian group of order six with (10, 10, 10) as the identity with respect to multiplication modulo 18.

It is important to record the following facts as, $18 = 9 \times 2 = 3^2 \times 2$ we see the only neutral elements of Z_{18} is 9 and 10, however 9 does not contribute to any nontrivial neutrosophic triplet, but 10 contributes 6 neutrosophic triplets which forms a group - group of neutrosophic triplets.

It is interesting to observe that in fact K is a cyclic group of order six.

Further we see none of the elements in Z_{18} which are multiples of three; do not contribute to neutrosophic triplets. To be more exact, 3, 6, 12 and 15 do not contribute to neutrosophic triplets.

Only 2, 4, 8, 10, 14 and 16 contribute to neutrosophic triplets with 10 as the neutral element 9 is only a neutral element which yields the trivial neutrosophic triplet (9, 9, 9).

Example 2.3. Let S = $\{Z_{54}, \times\}$ be the semigroup under product modulo 54. $54 = 3^3 \times 2$, $3^3 = 27$ is only odd.

The idempotents or neutral elements of Z_{54} are as follows.

$27 \times 27 = 27 \pmod{54}$ and $28 \times 28 \equiv 28 \pmod{54}$

are the only neutrals (idempotents of Z_{54}).

Now we find the neutrosophic triplets associated with the neutral element 27.

The only neutrosophic triplet group contributed by the neutral element 27 is (27, 27, 27).

Now we find the neutrosophic triplets associated with the neutral element 28.

(2, 28, 14) and (14, 28, 2) are neutrosophic triplets associated with the neutral element 28.

(4, 28, 34) and (34, 28, 4) are neutrosophic triplets of 28.

(8, 28, 44) and (44, 28, 8) are neutrosophic triplets of 28.,

(16, 28, 22) and (22, 28, 16) are neutrosophic triplets associated the neutral element 28.

(32, 28, 38) and (38, 28, 32) are neutrosophic triplets of 28.

(10, 28, 46) and (46, 28, 10) are neutrosophic triplets of the neutral element 28.

(20, 28, 50) and (50, 28, 20) are neutrosophic triplets of 28.

(40, 28, 52) and (52, 28, 40) are neutrosophic elements associated with the idempotents 28.

(26, 28, 26) and (28, 28, 28) are neutrosophic triplets.

Let B = {(28, 28, 28), (26, 28, 26), (40, 28, 52), (52, 28, 40), (20, 28, 50), (50, 28, 20), (10, 28, 46), (46, 28, 10), (32, 28, 38), (38, 28, 32), (22, 28, 16), (16, 28, 22), (8, 28, 44), (44, 28, 8), (4, 28, 34), (34, 28, 4), (2, 28, 14), (14, 28, 2)}.

It is easily verified B is a cyclic group - group neutrosophic triplet of order 18.

We see $\{Z_{18}, \times\}$ has a cyclic group - group neutrosophic triplet of order six $(18 = 2 \times 3^2)$.

Further $\{Z_{54}, \times\}$ has a cyclic group - group neutrosophic triplet of order 18 $(54 = 2 \times 3^3)$.

We see yet another example before we make some conjectures.

Example 2.4. Let $S = \{Z_{162}, \times\}$ be the semigroup under product modulo 162. The idempotents of Z_{162} are 81 and 82 as

$81 \times 81 \equiv 81 \pmod{162}$ and $82 \times 82 \equiv 82 \pmod{162}$.

The neutrosophic triplets associated with the neutral element 82 are (2, 82, 122) and (122, 82, 2) are neutrosophic triplets associated with the idempotent 82.

(4, 82, 142) and (142, 82, 4) are neutrosophic triplets of the neutral element 82.

(8, 82, 152) and (152, 82, 8) are neutrosophic triplets.

(16, 82, 76) and (76, 82, 16) are neutrosophic triplets associated with 82.

(32, 82, 58) and (58, 82, 32) are neutrosophic triplets associated with 82 are

(64, 82, 110) and (110, 82, 64) are neutrosophic triplets.

(128, 82, 136) and (136, 82, 128) are neutrosophic triplets related with the neutral element 82.

(94, 82, 94) and (82, 82, 82) are neutrosophic triplets.

(10, 82, 154) and (154, 82, 10) are neutrosophic triplets associated with the neutral element 82.

(20, 82, 158) and (158, 82, 20) are neutrosophic triplets associated with the idempotent 82.

(40, 82, 160) and (160, 82, 40) are neutrosophic triplets.

(80, 82, 80) is a neutrosophic triplet.

(14, 82, 14) is a neutrosophic triplet.

(28, 82, 88) and (88, 82, 28) are neutrosophic triplets.

(56, 82, 44) and (44, 82, 56) are neutrosophic triplets associated with the idempotents 82.

(112, 82, 22) and (22, 82, 112) are neutrosophic triplets.

(62, 82, 92) and (92, 82, 62) are neutrosophic triplet.

(124, 82, 46) and (46, 82, 124) are neutrosophic triplets.

(86, 82, 104) and (104, 82, 86) are neutrosophic triplets.

(26, 82, 134) and (134, 82, 26) are neutrosophic triplets.

(52, 82, 148) and (148, 82, 52) are neutrosophic triplets.

(34, 82, 34) is a neutrosophic triplet.

(68, 82, 98) and (98, 82, 68) are neutrosophic triplets.

(6, 82, 66) and (66, 82, 6) are neutrosophic triplets.

Clearly (12, 82, 114) and (114, 82, 12) are neutrosophic triplets.

Further (24, 82, 138) and (138, 82, 24) are neutrosophic triplets.

(48, 82, 150) and (150, 82, 48) are neutrosophic triplets associated with the neutral element 82.

(96, 82, 156) and (156, 82, 96) are neutrosophic triplets.

(30, 82, 78) and (78, 82, 30) are neutrosophic triplets.

(60, 82, 120) and (120, 82, 60) are neutrosophic triplets.

The reader is left with the task of finding the total number of neutrosophic triplets associated with the neutral element 82.

Conjecture 2.1. Finding the number of neutrosophic triplets in case of Z_n where $n = 3^t \times 2$, $t \geq 1$ happens to be a open conjecture.

Further it is conjectured if m is the number of neutrosophic triplet associated with $(\frac{n}{2} + 1)$ then can we say 6 divides m?

Next we proceed onto work with specific order of n is Z_{2n}.

Example 2.5. Let $S = \{Z_{210}, \times\}$ be the semigroup. We find the idempotents of Z_{210}. We see $210 = 2\times 3 \times 5 \times 7$.

The idempotents of Z_{210} are

$15 \times 15 \equiv 15 \pmod{210}$, $21 \times 21 = 21 \pmod{210}$,

$36 \times 36 = 36 \pmod{210}$, $70 \times 70 \equiv 70 \pmod{210}$,

$85 \times 85 \equiv 85 \pmod{210}$, $91 \times 91 = 91 \pmod{210}$,

$106 \times 106 = 106 \pmod{210}$, $120 \times 120 = 120 \pmod{210}$,

$105 \times 105 \equiv 105 \pmod{210}$, $126 \times 126 \equiv 126 \pmod{210}$,

$141 \times 141 \equiv 141 \pmod{210}$, $175 \times 175 = 175 \pmod{210}$,

$190 \times 190 = 190 \pmod{210}$ and $196 \times 196 \equiv 196 \pmod{210}$.

There are 14 idempotents. We can have 14 sets of distinct neutrosophic triplets associated with these 14 neutral elements of Z_{210}.

(2, 106, 158) and (158, 106, 2) are neutrosophic triplets associated with the neutral element 106.

(4, 106, 184) and (184, 106, 4) are neutrosophic triplets.

(8, 106, 92) and (92, 106, 8) are neutrosophic triplets.

(16, 106, 46) and (46, 106, 16) are neutrosophic triplets.

(32, 106, 128) and (128, 106, 32) are neutrosophic triplets.

(64, 106, 64) and (106, 106, 106) are neutrosophic triplets.

(3, 141, 1171) and (117, 141, 3) are neutrosophic triplets.

(9, 141, 39) and (39, 141, 9) are neutrosophic triplets.

(27, 141, 153) and (153, 141, 27) are neutrosophic triplets.

(81, 141, 81) and (141, 141,141) are neutrosophic triplets.

(5, 85, 185) and (185, 85, 5) are neutrosophic triplets.

(25, 85, 205) and (205, 85, 25) are neutrosophic triplets.

(125, 85, 125) and (85, 85, 85) are neutrosophic triplets.

(6, 36, 6) and (36, 36, 36) are neutrosophic triplets.

(7, 91, 133) and (133, 91, 7) are neutrosophic triplets.

(49, 91, 49) and (91, 91, 91) are neutrosophic triplets.

(10, 190, 40) and (40, 190, 10) are neutrosophic triplets.

(100, 190, 130) and (130, 190, 100) are neutrosophic triplets.

(160, 190, 160) and (190, 190, 190) are neutrosophic triplets.

(12, 36, 108) and (108, 36, 12) are neutrosophic triplets.

(144, 36, 114) and (114, 36, 144) are neutrosophic triplets.

(48, 36, 132) and (132, 36, 48) are neutrosophic triplets.

(156, 36, 186) and (186, 36, 156) are neutrosophic triplets.

(192, 36, 192) and (36, 36, 36) are neutrosophic triplets.

(14, 196, 14) and (196, 196, 196) are neutrosophic triplets.

(18, 36, 72) and (72, 36, 18) are neutrosophic triplets.

(20, 190, 20) is a neutrosophic triplet.

(24, 36, 54) and (54, 36, 24) are neutrosophic triplets.

(156, 36, 186) and (186, 36, 156) are neutrosophic triplets.

(174, 36, 174) is a neutrosophic triplet.

(28,196, 112) and (112, 196, 28) are neutrosophic triplets.

(154, 196, 154) is a neutrosophic triplet.

(30, 120, 60) and (60, 120, 30) are neutrosophic triplets.

(120, 120, 120) is a neutrosophic triplet.

(35, 175, 35) is a neutrosophic triplet.

(200, 190, 170) and (170, 190, 200) are neutrosophic triplets.

(50, 190, 50) is a neutrosophic triplet.

(150, 120, 180) and (180, 120, 150) are neutrosophic triplets.

(42, 126, 168) and (168, 126, 42) are neutrosophic triplets associated with the neutral element 126.

(84, 126, 84) and (126, 126, 126) are neutrosophic triplets of the idempotent 126.

The reader is left with the task of finding the number of neutrosophic triplets associated with all the neutral elements of Z_{210}.

Further it is pertinent to record that when the number of primes in the n (of Z_n) is large so is the number of neutral elements (idempotents of Z_n).

We see the Z_{2n} where n is not a prime has several neutral (idempotents) elements.

But when 'n' is an odd prime Z_{2n} has only two idempotents or neutral elements viz. $\frac{n+1}{2}$ and $\frac{n}{2}$. It is further observed that $\frac{n}{2}$ contributes only to trivial neutrosophic triplet, thus only $\frac{n+1}{2}$ contributes to several nontrivial neutrosophic triplets.

The results related with them are obtained.

Now we proceed on to work with elements from Z_{3p}, p may be a prime or an odd number not divisible by three.

Example 2.6. Let S = $\{Z_{33}, \times\}$ be the semigroup under product modulo 33.

The neutral elements or idempotents of Z_{33} are

$12 \times 12 \equiv 12 \pmod{33}$ and $22 \times 22 \equiv 22 \pmod{33}$.

The neutrosophic elements associated with 12. That is (3, 12, 15) and (15, 12 30) are neutrosophic triplets of the idempotent 12.

(9, 12, 27) and (27, 12, 9) are neutrosophic triplets of the idempotent 12.

(18, 12, 30) and (30, 12,18) are neutrosophic triplets of the neutral element 12.

(21, 12, 21) is a neutrosophic triplet.

(12, 12, 12) is also the trivial neutrosophic triplet.

We see the neural element 22 does not contribute to any non trivial triplet.

6 is the only non unit which does not contribute to neutrosophic triplet in Z_{33}.

The observations are important

i) As in case of Z_{2p}, p an odd prime we see in case of Z_{3p}, p a prime the number of neutral elements are only two one of them is just a trivial neutral element where as the other gives a number of nontrivial triplets.

ii) Incase of Z_{33} we see the element 6 is a zero divisor and it does not contribute to any neutrosophic triplets.

This is a special feature of Z_{33} however will this type of observation be true in case of Z_{3p}, p a prime.

We observe in case of Z_{15} the two neutral elements are 6 and 10 are both non trivial neutral elements.

Here both 6 and 10 non trivial neutral elements for (5, 10, 5), (10, 10, 10), (3, 6, 12), (12, 6, 3), (9, 6, 9) and (6, 6, 6) are the total number of neutrosophic triplets.

For S = $\{Z_{33}, \times\}$ the neutral elements are 12 and 22. 22 is a trivial neutral element and 12 gives all the nontrivial

neutrosophic triplets some of which are (3, 12, 15), (15, 12, 3), (9, 12, 27), (27, 12, 9) and so on.

Likewise we have found the neutrals of Z_{21}, Z_{39}, Z_{51}, Z_{57}, z_{69}, Z_{87}, Z_{93}, Z_{111} and are tabled in the following.

Semigroup Z_{3p}, p a prime	**Neutrals associated with Z_p**	
Z_{15}	6	10
Z_{33}	12	22
Z_{39}	13	27
Z_{51}	18	34
Z_{57}	19	39
Z_{69}	24	46
Z_{87}	30	58
Z_{93}	31	63
Z_{111}	37	75
Z_{123}	42	82
Z_{129}	43	87
Z_{141}	48	94
Z_{159}	54	106
Z_{177}	60	119
Z_{183}	61	123

and so on.

It is interesting to make the following observations.

i) If the neutral elements are considered for Z_{15} then $1+ 5 = 6$ so the idempotents are $(5 + 1)$ and 2×5.

Consider Z_{33}, $3+3 = 6$ the neutral elements of Z_{33} are 12 and 22 and $11 + 1 =12$ and $11 \times 2 = 22$.

For Z_{39}, $3 + 9 = 12$, $1 + 2 = 3$. The neutral elements of Z_{39} are 13 and $27 =2 \times 13 + 1$.

For this Z_{57}, $5 + 7 = 12$; $1 + 2 = 3$ and the neutral elements of Z_{57} are 19 and $39 = 2 \times 18 + 1$.

For Z_{93}, $9 + 3 = 12$, $1 + 2 = 3$ and neutral elements of Z_{93} are 31 and $63 = 31 \times 2 + 1$.

Thus we see if an Z_{3p} the sum of 3p add up to 3 then the neutral elements are p and 2p + 1.

If the sum of the elements of 3p adds upto 6 then the neutral elements are p + 1 and 2p.

Thus it is conjectured as follows.

Conjecture 2.2. Let Z_{3p}, $p > 5$, p a prime be the semigroup under product if the sum of all the digits in 3p adds to 3 then the two neutral elements of Z_{3p} are p and 2p + 1.

If the sum of the digits of 3p adds to 6 then the two neutral elements of Z_{3p} are p + 1 and 2p.

It is noted the sum of the digits of 3p for all primes p adds upto either 3 or 6 only.

We give some more examples of this conjecture.

Example 2.7. Let S = {Z_{393}, ×} be the semigroup. Sum of 393 is 3 + 9 + 3 = 6 and the neutral elements of Z_{393} are 132 and 272. Hence the conjecture is true for $Z_{393} = Z_{3p} = Z_{3 \times 131}$.

Example 2.8. Let S = {Z_{597}, ×} be the semigroup. 597 = 5 + 9 + 7 = 21; 2 + 1 = 3.

The neutral elements of Z_{597} are 199 and 399.

The conjecture is true in case of Z_{597}.

Now consider with p × q where p is 5 and q is a prime.

Example 2.9. Let S = {Z_{35}, ×} be the semigroup. The neutral elements of Z_{35} are 15 and 21.

Example 2.10. Consider S = {Z_{55}, ×} be the semigroup. The neutral elements of S are 11 and 45.

We observe in this case when Z_{pq}, p and q two distinct primes can define the notion of pseudo primitive element.

Recall if {Z_p, ×,+} be the a field of characteristics p, p a prime then we can define primitive elements in Z_p.

An element $x \in Z_p \setminus \{0\}$ is called a primitive element of Z_p if $x^p = x$ or $x^{p-1} = 1$.

We see some examples of them.

Let S ={Z_{11}, ×, +} be the finite field of order 10.

$Z_{11} \setminus \{0\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

We see for $2 \in Z_{11}$, $2^{10} = 1$ or $2^{11} = 2048 \pmod{11}$

$= 2 \pmod{11}$.

Thus 2 is a primitive element of Z_{11}.

Consider $3 \in Z_{11}$; $3^5 = 243 \pmod{n} = 1 \pmod{11}$ or $3^6 \equiv 3$ so 3 is not a primitive element of Z_{11}.

Consider $4 \in Z_{11}$, we see

$4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024 \equiv 1 \pmod{11}$

So 4 is not a primitive element of Z_{11}

Consider $5 \in Z_1$, $5^2 = 3 \pmod{11}$

$5^4 = 9 \pmod{11}$ $5^6 = 5 \pmod{11}$.

So 5 is not a primitive element of Z_{11}.

For we see $5 \times 5 \times 5 \times 5 \times 5 = 1 \pmod{11}$.

Let $6 \in Z_{11}$,

$6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 1 \pmod{11}$.

That is $6^{10} = 1$ and $6^{11} = 6$ so 6 is again a primitive element of Z_{11}.

$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \equiv 1 \pmod{11}$.

$7^{10} \equiv 1$ and $7^{11} = 7$ so 7 is again a primitive element of Z_{11}.

$8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^{10} \equiv 1 \pmod{11}$ and $8^{11} \equiv 8 \pmod{11}$.

$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \equiv 9 = 1 \pmod{11}$ and $9^{11} = 9 \pmod{11}$.

$10 \times 10 = 1 \pmod{11}$.

We see Z_{11} has 5 nontrivial primitive elements.

Now we proceed onto describe and define the pseudo primitive elements of Z_{2n} where n is a prime.

Definition 2.1. Let $S = \{Z_{2p}, \times\}$; p an odd prime. Let $K = \{2, 4, \ldots, 2p - 2\}$ be the collection of all even elements of Z_{2p}. $p + 1 \in K$ is such that $x \times (p + 1) = p + 1$ for all $x \in K$. Further there exists a $y \in K$ such that $y^{p-1} = p + 1$.

We call this y as the pseudo primitive element of $K \subseteq Z_{2p}$.

We will first show this by some examples.

Example 2.11. Let $S = \{Z_{74}, \times\}$ be the semigroup under multiplication modulo 74. $74 = 2 \times 37$; 37 is the odd prime. $K = \{2, 4, 6, 8, 10, 12, \ldots, 36, 38, 40, 42, \ldots, 70, 72\} \subseteq S$. $38 \in Z_{74}$ is such that $38 \times x = x$ for all $x \in K$.

$2 \in K$ is such that $2^{20} = 38$.

Consider $4 \in K$, $4^{10} = 38$. The reader is left with the task of finding the pseudo primitive element of Z_{2p}.

Example 2.12. Let $S = \{Z_6, \times\}$ be the semigroup under product $K = \{2, 4\}$ and $2 \times 4 = 2$ but $2^2 = 4 = 2^{3-1}$ so 2 is the pseudo primitive element of K.

Example 2.13. Let S = $\{Z_{10}, \times\}$ be the semigroup under product K = $\{2, 4, 6, 8\} \subseteq Z_{10}$; $2 \in K$ is the pseudo primitive element of K as $2 \times 2 \times 2 \times 2 = 6 = 2^4 = 2^{5-1}$ is the pseudo primitive element of K is verified $4 \times 4 = 6$ so 4 is not a pseudo primitive element of K.

Consider $8 \in K$, $8 \times 8 \times 8 \times 8 = 6$ so 8 is also a pseudo primitive element of K.

Example 2.14. Let S = $\{Z_{22}, \times\}$ be the semigroup under product. K = $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \subseteq S$ is a group in fact a cyclic group of order 10 with 12 as the multiplicative identity. Clearly $2 \in K$ is such that $2^{10} = 12$. $4 \in K$ is not a pseudo primitive element of K.

2 is a pseudo primitive element of K.

Interested reader can find other pseudo primitive elements of this K.

Now a natural question arises; can Z_{3p}, p an odd prime have the notion of pseudo primitive elements.

To this effect we study the following examples.

Example 2.15. Let S = $\{Z_{15}, \times\}$ be the semigroup under product modulo 15. Let P = $\{3, 6, 9, 12\} \in S$ is such that 6 is the identity element of P.

For $3 \times 6 \equiv 3 \pmod{15}$, $6 \times 6 = 6 \pmod{15}$,

$9 \times 6 = 9 \pmod{15}$ and $12 \times 6 = 12 \pmod{15}$.

$3 \in P$ is such that $3^4 = 6 = 3^{5-1}$. Thus 3 is the pseudo primitive element of P. $9 \in P$ is such that $9^2 = 6$ so 9 is not a pseudo primitive element of P.

$12 \in P$ is such that

$12 \times 12 = 9 \pmod{22}$, $9 \times 12 = 3 \pmod{22}$ and

$3 \times 12 = 6 \pmod{22}$.

So $12^4 = 12^{5-1} = 6$ is the pseudo primitive element of P.

Example 2.16. Let $S = \{Z_{35}, \times\}$ be the semigroup under product modulo 35. We now find for $R = \{5, 10, 15, 20, 25, 30\} \subseteq Z_{35}$ has pseudo primitive elements. For $T = \{7, 4, 21, 28\} \subseteq Z_{35}$; we find out whether T has identity. We find the multiplicative tables of R and T.

$\times$	5	10	15	20	25	30
5	25	15	5	30	20	10
10	15	30	10	25	5	20
15	5	10	15	20	25	30
20	30	25	20	15	10	5
25	20	5	25	10	30	15
30	10	20	30	5	15	25

Clearly 15 is the identity element of R. 5 generates R and $5^6 = 15$.

The table for T is as follows.

$\times$	7	14	21	28
7	14	28	7	21
14	28	21	14	7
21	7	14	21	28
28	21	7	28	14

21 is the identity of this cyclic group with 7 as one of the generators. 28 and 7 are the pseudo primitive elements of T.

For $7 \times 7 = 14$. $7^3 = 14 \times 7 = 28$. $7^4 = 28 \times 7 = 21$ thus $7 \times 7 \times 7 \times 7 = 21$ so 7 is a pseudo primitive element of T.

The pseudo primitive element of R are 5 and 10 are some of the pseudo primitive elements of R.

For $5 \times 5 = 25 \pmod{35}$ $5^3 = 25 \times 5 = 125 \pmod{35} = 20$.

$$5^4 = 20 \times 5 = 100 = 30 \pmod{35}$$

$$5^5 = 30 \times 5 = 150 \pmod{35} = 10.$$

Thus $5^6 = 10 \times 5 = 50 \equiv 15 \pmod{35}$.

So 5 is a proved to be a pseudo primitive element of R as $5^7 = 5 \pmod{35}$.

In view of all the facts we have the following theorem.

Theorem 2.1. *Let $S = \{Z_{p\ q}, \times\}$ be the semigroup under product; p and q two distinct primes*

i) *$R = \{p, 2p, \ldots, (q-1)p\}$ is a cyclic group of order $q - 1$ under product modulo pq.*

ii) *$P = \{q, 2q, \ldots, (p-1)q\}$ is a cyclic group of order $p - 1$ under product modulo pq.*

iii) *Both R and P has pseudo primitive elements.*

Proof is direct and hence left as an exercise to the reader.

Now we find if $S = \{Z_{105}, \times\}$ is taken as a semigroup under product modulo 105. $105 = 3 \times 5 \times 7$ we just test for the existence of pseudo primitive elements.

The collection $S_1 = \{3, 6, 9, 12, 15, 18, 21, 24, \ldots, 102\}$,

$S_2 = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100\}$ and

$S_3 = \{7, 14, 21, 28,35, 42, 49, 56, 63, 70, 77, 84, 91, 98\}$ are subsets of S.

The reader is left with the task of finding the structure of S_1, S_2 and S_3.

If $P_1 = \{15, 30, 45,60, 75, 90\}$,

$P_2 = \{21, 42, 63, 84\}$ and

$P_3 = \{35, 70\}$ be the collection. Do these form a group or semigroup under product modulo 105.

The table for P_3

×	35	70
35	70	35
70	35	70

P_3 is a cyclic group of order two 70 acts as the multiplication identity. $35 \times 35 \equiv 70 \pmod{15}$. The table for P_2 is as follows.

×	21	42	63	84
21	21	42	63	84
42	42	84	21	63
63	63	21	84	42
84	84	63	42	21

We see P_2 is again a cyclic group of order 4 with 21 as the identity. $42^4 = 21$. So 42 is the pseudo primitive element of P_2. $42^5 = 42$, $42^{5-1} = 42^4 = 21$.

Now we give the table of P_1 in the following.

×	15	30	45	60	75	90
15	15	30	45	60	75	90
30	30	60	90	15	45	75
45	45	90	30	75	15	60
60	60	15	75	30	90	45
75	75	45	15	90	60	30
90	90	75	60	45	30	15

Clearly P_1 is a group with 15 as the identity under product modulo 105.

$45 \in P_1$ is such that $45 \times 45 \times 45 \times 45 \times 45 \times 45 \equiv 15$ that is $45^{7-1} = 45^6 = 15$ is the pseudo primitive element of P_1.

Thus we wish to make the following conjecture.

Conjecture 2.3. Let $S = \{Z_{pqr}, \times\}$ where p, q and r three distinct primes.

Let $P_1 = \{pq, 2pq, .., (r-1)pq\}$,

$P_2 = \{qr, 2qr, \ldots, (p-1)\, qr\}$ and

$P_3 = \{rp, 2rp, \ldots, (q-1)\, rp\}$ be the collection from S.

i) All the three sets P_1, P_2 and P_3 are cyclic groups.
ii) Every P_i has pseudo primitive elements, $1 \leq i \leq 3$.

We want to find the structure when Z_{pqrs} where p, q, r and s are four distinct primes.

First we illustrate this situation by some examples.

Example 2.17. Let $S = \{Z_{210}, \times\}$ be the semigroup under product modulo 210.

Clearly $210 = 2 \times 3 \times 5 \times 7$ the product of four distinct primes.

Consider B_1 = {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156, …, 204}.

B_2 = {10, 20, 30, 40, 50,60, 70, 80, 90, …, 200},

B_3 = {14, 28, 42, 56, 70, 84, 98, …, 182, 196},

B_4 = {15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195},

B_5 = {21, 42, 63, 84, 105, 126, 147, 168, 189}

B_6 = {35, 70, 105,140, 175}

B_7 = {30, 60, 90, 120, 150, 180},

B_8 = {42, 84, 127, 168} and B_9 ={70, 140} are groups under product modulo 210.

We see B_9 is a cyclic group of order two.

×	70	140
70	70	140
140	140	70

We now find the table for B_8 which is as follows.

×	42	84	126	168
42	84	168	42	126
84	168	126	84	42
126	42	84	126	168
168	126	42	168	84

Clearly B_8 is again a cyclic group under product modulo 210 with 126 as the identity element.

B_8 also has 42 to be its pseudo primitive element.

Now we consider the table of B_7 in the following and test for its properties.

×	30	60	90	120	150	180
30	60	120	180	30	90	150
60	120	30	150	60	180	90
90	180	150	120	90	60	30
120	30	60	90	120	150	180
150	90	180	60	150	30	120
180	150	90	30	180	120	60

We see B_7 is also a cyclic group of order six generated by 150 as $150^6 = 120$, 120 is the identity of B_7.

Further the pseudo primitive element of B_7 is 150. In fact B_7 may have other pseudo primitive elements.

Now we find the table of B_6 and enumerate the special features enjoyed by it.

×	35	70	105	140	175
35	175	140	105	70	35
70	140	70	0	140	70
105	105	0	105	0	105
140	70	140	0	70	140
175	35	70	105	140	175

We see B_6 is even not closed with respect to product modulo 210. Infact $B_6 \cup \{0\}$ can yield only a monoid with 175 as its multiplicative identity.

The reader is left with the task of finding the algebraic structure enjoyed by $B_1, B_2, \ldots, B_5$.

However one is interested in finding the properties enjoyed by the pseudo primitive elements of sets B_i and B_j ($i \neq j$) for we know in case $i = j$ they only act as part of a group.

Let us now consider the pseudo primitive elements of B_7 and B_8. The pseudo primitive element of B_8 is 42 and that of B_7 is 150. We see $42 \times 150 \equiv 0 \pmod{210}$.

In fact they form orthogonal sets.

At this juncture we are forced to conclude if B_i's form cyclic groups then $B_i \times B_j = \{0\}$ if $i \neq j$.

We still find it difficult to find the algebraic structure enjoyed by B_1, B_2 and so on for they are generated singly by a prime number which is a factor of 210.

Such study is both innovative and interesting so left as an exercise to the reader.

We now find the pseudo primitive elements when in Z_n $n = p^2 q^2 r$, r, p and q three distinct primes.

This will first be illustrated by some examples.

Example 2.18. Let $S = \{Z_{180}, \times\}$ be the semigroup under product modulo 180.

Let $B_1 = \{2, 4, 6, 8, \ldots, 178\}$,

$B_2 = \{4, 8, 12, 16, 20, 24, \ldots, 176\}$,

$B_3 = \{3, 6, 9, 12, 15, 18, \ldots, 177\}$,

$B_4 = \{9, 18, 27, 36, 45, 54, \ldots, 171\}$,

$B_5 = \{12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168\}$,

$B_6 = \{18, 36, 54, 72, 90, 108, 126, 144, 162\}$,

$B_7 = \{36, 72, 108, 14\}$,

$B_8 = \{10, 20, 30, 40, 50, \ldots, 160, 170\}$,

$B_9 = \{20, 40, 60, 80, 100, 120, 140, 160\}$,

$B_{10} = \{15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165\}$,

$B_{11} = \{45, 90, 135\}$,

$B_{12} = \{30, 60, 120, 150\}$,

$B_{13} = \{12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168\}$ and

$B_{14} = \{60, 120\}$ are the subsets.

We can find the algebraic structure enjoyed by these sets.

The reader is expected to find the algebraic structure enjoyed by them.

We conjecture the following:

Conjecture 2.4. Let $\{Z_n, \times\} = S$ be the semigroup under product n, let $n = p_1^2 p_2^2 q$ where p_1, p_2 and q are distinct primes.

i) Prove only a few of the sets generated by $\langle p_1\rangle$, $\langle p_2\rangle$, $\langle q\rangle$, $\langle p_1^2\rangle$, $\langle p_2^2\rangle$, $\langle p_1 q\rangle$, $\langle p_2 q\rangle$, $\langle p_1^2 q\rangle$, $\langle p_2^2 q\rangle$, $\langle p_1 p_2 q\rangle$, $\langle p_1 p_2^2 q\rangle$, $\langle p_1^2 p_2 q\rangle$, $\langle p_1^2 p_2\rangle$ and $\langle p_2^2 p_1\rangle$ are cyclic groups.

ii) Characterize those cyclic groups which contain pseudo primitive elements.

iii) Prove if x_1 and x_2 are two pseudo primitive elements of two distinct groups then they are always orthogonal.

Now in the following we propose the probable applications of these new structures.

Let Z_n where n is a composite number be the semigroup under product modulo n.

We see the groups which have a pseudo primitive elements can generate algebraic codes in which we need only to adjoin the zero element. Further it is pertinent to keep on record that these two codes if we take the same length say a (n, k) code then they will certainly be orthogonal.

This situation will be exhibited by some examples.

Example 2.19. Let $S = \{Z_{pq}, \times\}$ be the semi group under product modulo pq where p and q are two distinct odd primes.

To me more specific let us assume p = 7 and q = 11.

We first find the neutrals or idempotents of $Z_{pq} = Z_{77}$.

The idempotents or neutrals of Z_{77} are 56 and 22 are the only idempotents of Z_{77}.

Let $B_1 = \{7, 14, 21, 28, 35, 42, 49, 56, 63, 70\}$ and

$B_2 = \{11, 22, 33, 44, 55, 66\}$ be the two sets.

We describe the tables of them. The table for B_2 is as follows.

$\times$	11	22	33	44	55	66
11	44	11	55	22	66	33
22	11	22	33	44	55	66
33	55	33	11	66	44	22
44	22	44	66	11	33	55
55	66	55	44	33	22	11
66	33	66	22	55	11	44

It is easily verified B_2 is a group under product modulo 77 of order 6 infact a cyclic group with 22 as its identity and 33 $\in B_2$ is the generator of B_2 as $(33)^6 = 22$.

Suppose we are interested in finding the algebraic codes of length 5 built using $B_2 \cup \{0\}$.

Let $C_2 = (5,2)$ algebraic codes with entries from $B_2 \cup \{0\}$.

Clearly $o(C_2) = 7^5$.

Any code word $x = (a_1, a_2, a_3, a_4, a_5)$ where

$a_i \in \{B_2 \cup \{0\}; 1 \leq i \leq 5$.

Now we give the table for B_1 in the following.

$\times$	7	14	21	28	35	42	49	56	63	70
7	49	21	70	42	14	63	35	7	56	28
14	21	42	63	7	28	49	70	14	35	56
21	70	63	56	49	42	35	28	21	14	7
28	42	7	49	14	56	21	63	28	70	35
35	14	28	42	56	70	7	21	35	49	63
42	63	49	35	21	7	70	56	42	28	14
49	35	70	28	63	21	56	14	49	7	42
56	7	14	21	28	35	42	49	56	63	70
63	56	35	14	70	49	28	7	63	42	21
70	28	56	7	35	63	14	42	70	21	49

Clearly B_L is a cyclic group of order 10 with 56 as its identity. $7^{10} = 56$ is a pseudo primitive element of B_1.

Now let $C_1 = (5, 2)$ be the algebraic code of length n with 3 message symbols and 2 check symbols with entries from ($B_1 \cup \{0\}$).

We see $o(C_1) = 10^5$.

Further it is clearly if $x \in C_2$ and $y \in C_1$ then

$x \times y = (0, 0, 0, 0, 0)$.

Let x = (11, 0, 22, 66, 55) $\in C_2$ and y = (7, 49, 63,70, 14) $\in C_1$ we see x $\times$ y = (0, 0, 0, 0, 0).

It is easily proved C_1 is a dual code of C_2 and vice versa.

By this method without any difficulty we can arrive at dual code from the collection Z_{17}.

In general first we make the following theorem.

Theorem 2.2. *Let S = $\{Z_{pq}, \times\}$ be the semigroup of order pq where p and q are two distinct primes;*

i) *$B_1 = \{p_1, 2p, \ldots, (q-1)p, 0\}$ is a field of order q under + and $\times$ modulo pq.*

ii) *$B_2 = \{q, 2q, 3q, \ldots, (p-1)q, 0\}$ is again a field of order p under + and $\times$ modulo pq.*

iii) *C_1 =(n, k) the code of length n with k message symbols with entries from B_1.*

iv) *C_2 = (n, k) is the code of length n with k messages with symbols from B_2.*

v) *C_1 is orthogonal with C_2.*

Proof is left as an exercise for the theorem.

Now we proceed onto test the number of orthogonal codes in case of Z_{pqr} where p, q and r are three distinct primes by some examples.

Example 2.20. Let S = $\{Z_{165}, \times\}$ be the semigroup under product or a ring of modulo integers.

$165 = 3 \times 5 \times 11$ is the product of three primes.

Let $B_1 = \{15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 0\}$ is a field of order 11 under product modulo 165.

$B_2 = \{33, 66, 99, 132, 0\}$ is again a field of order 5.

$B_3 = \{55, 110, 0\}$ is a field of order 3. All these are fields. If we build (n, k) codes C_1, C_2 and C_3 using B_1, B_2 and B_3 respectively it can be easily proved C_i is orthogonal with C_j, $i \neq j$, $1 \leq i, j \leq 3$.

Example 2.21. Let $S = \{Z_{3289}, +, \times\}$ be a ring of modulo integers, clearly $3289 = 11 \times 13 \times 23$.

Consider $B_1 = \{0, 143, 286, 429, \ldots, 3146\}$,

$B_2 = \{0, 299, 598, 897, \ldots, 2990\}$ and

$B_3 = \{0, 253, 506, 759, 1012, \ldots, 3036\}$ be the fields of order 23, 11 and 13 respectively.

If C_1, C_2 and C_3 are algebraic codes using the fields B_1, B_2 and B_3 we see these codes are mutually orthogonal.

In view of all these we can put for the following result. If some researcher is interested in constructing a set of t dual codes of same length then following procedure can be adopted.

Take $F = \{Z_{p_1 p_2 \ldots p_t}, \times, +\}$ be the ring of modulo integers, where $p_1, p_2, \ldots, p_t$ are t distinct primes.

Let $B_1 = \{p_1 p_2, \ldots p_{t-1}, 2(p_1 \ldots p_{t-1}), \ldots (p_t - 1)(p_1 p_2 \ldots p_{t-1}), 0\}$ is a field with p_t number of elements in it.

$B_2 = \{(p_1p_2\ldots p_{t-2}p_t), 2(p_1p_2 \ldots p_{t-2}p_t), \ldots, (p_t - 1)(p_1p_2 \ldots p_{t-2}p_t),$

$B_3 = \{p_1p_2 \ldots p_{t-3}p_{t-1}, p_t, 2(p_1 p_2 \ldots p_{t-3} p_t), \ldots, (p_{t-2} - 1) (p_1p_2 \ldots p_{t-3} p_{t-1} p_t)$ are fields of order p_{t-1} and p_{t-2} respectively. Proceed in the same way

$B_t = \{p_2 \ldots p_t, 2(p_2 \ldots p_t), \ldots, (p_1 - 1) (p_2 \ldots p_t), 0\}$ is a field of order p_1.

Now we see using $Z_{p_1p_2\ldots p_t}$, we can build t number of distinct fields we use these t distinct fields to construct algebraic codes of same length say $C_1, C_2, \ldots, C_t$ associated respectively with $B_1, B_2, \ldots, B_t$.

It is easily verified that these t codes $C_1, C_2, \ldots, C_t$ are such that they are mutually orthogonal, that is $C_i \times C_j = \{(0, 0, \ldots, 0)\}$ if $i \neq j$, $1 \leq i, j \leq t$.

So this study of modulo integers has led to the construction of dual codes. Now we make the following conjecture.

Conjecture 2.5. Let $S = \{Z_{p_1p_2\ldots p_t}, +, \times\}$ be the ring of modulo integers, ($p_1, p_2, \ldots, p_t$; t distinct primes)

i) There are atleast t distinct fields in S.

ii) These fields has atleast t number of pseudo primitive elements associated with each field such that their product is {0}.

iii) These t number of fields can be used to construct t number of algebraic codes $C_1, \ldots, C_t$ of same length and they are mutually orthogonal.

Now we after this simple deviation proceed onto study neutrosophic triplets and can we have algebraic codes of neutrosophic triplets. The answer is a big no as the triplets are only groups under product and under + they are not even closed.

So such notions cannot be established.

Now we proceed onto suggest some problems to the reader some of which are difficult and some of them are just exercise.

Problems

1. Let S = $\{Z_{5063}, \times\}$ be the semigroup under product modulo 5063.

   a) Find the neutral elements of S.

   b) Using those neutral elements construct the neutrosophic triplets.

   c) Prove the neutrosophic triplets associated with each of the neutral element is a cyclic group under product.

   d) Show these collections are such that their product is {(0, 0, 0)}.

2. Let $S_1 = \{Z_{391}, \times\}$ be the semigroup under product modulo 391.

Study questions (a) to (d) of problem (1) for this S_1.

3. Let S = $\{Z_{9361}, \times\}$ be the semigroup under product modulo 9361.

   i) How many idempotents or neutral elements exist in Z_{9361}?

   ii) Find the classical group of neutrosophic triplet groups associated with each of the neutrals.

   iii) If G_i s are the classical groups.

      a) Prove each G_i is cyclic.
      b) Prove $G_i \times G_j = \{(0, 0, 0)\}$, if $i \neq j$.

   iv) Obtain any other special features enjoyed by the classical group of neutrosophic triplets.

4. Let P = $\{Z_{385}, \times\}$ be the semigroup under product modulo 385.

   i) How many neutrals or idempotents are in P?

   ii) How many of these neutrals contribute to neutrosophic triplet groups which are classical groups?

   iii) Let T = {35, 70, 105, 140, 175, 210, 245, 280, 315, 350} be collection of elements.

      a) Is T a group?
      b) Find the identity if T is group or a monoid.
      c) Does T contain pseudo primitive elements?

d) Can T contribute to neutrosophic triplet groups?

iv) How many cyclic groups exist in P?

v) Compare the S of problem 3 with this T.

5. Let $S_2 = \{Z_{4256}, \times\}$ be the semigroup under product modulo 4356.

i) Study questions (i) to (iv) of problem 3 for this S_2.

ii) Does S_1 of problem 3 or S_2 contain more number neutrals?

iii) Which collection from S_1 of problem 3 or S_2 yield more number of nontrivial neutrosophic triplet groups?

iv) Obtain the similarities and dissimilarities between S_1 and S_2.

6. Let $W = \{ Z_{2^4 3^3 5^2}, \times\}$ be the semigroup under product modulo $2^4 3^3 5^2$

i) Compare this W with S_1 of problem 4 and S_2 of problem 5.

ii) Which has maximum number of neutrals W or S_1 of problem 4 or S_2 of problem 5?

iii) Which of the semigroups W or S_1 or S_2 has maximum number of cyclic groups?

7. What are the special features associated with pseudo primitive elements of Z_n.

8. Can we claim that all pseudo primitive elements of $G \subseteq Z_n$ are idempotents or neutrals of Z_n?

9. Let $S = \{Z_{330}, \times, +\}$ be the ring of integers modulo 330.

   i) Find all neutrals of S.

   ii) Find all subsets of S which are finite fields.

   iii) Find all pseudo primitive elements of those finite fields.

   iv) If C_i are the algebraic codes of the finite fields F_i (for i = 1, 2, …, t) of same length prove they are orthogonal with each other.

   v) Why we cannot build codes using neutrosophic triplet groups? Justify your claim.

   vi) Obtain all special feature associated with this S.

10. Can we prove, $\{Z_n \times, +\}$ will give more number of orthogonal codes if $n = p_1, \ldots, p_t$, t distinct primes if t is large?

11. Compare the situation in problem (10) for $P = \{Z_{2.3.5.7.1.1.13}, \times, +\}$ and $R = \{Z_{43.53.13.17.11.7.5.23.41}, +, \times\}$ the ring of modulo integers.

    a) Which has more number of orthogonal codes P or R?

    b) Which ring P or R has more number of idempotents or neutrals?

c) Which has more number of neutrosophic triplet groups?

d) Analyse the differences and similarities of P and R.

12. Show the number of neutrals which can contribute to classical groups is of large size in case of S = $\{Z_n, \times / n = p_1 \ldots p_t\}$ than W = $\{Z_m, \times / m = p_1^{t_1} \ldots p^{t_s}, t_i > 1\}$.

i) Can we say S has more number of classical groups of neutrosophic triplet groups?

ii) Is it true W has more number of classical group of neutrosophic triplet groups?

iii) Which of the semigroups S or W will produce more number of finite fields?

iv) Find the similarities and dissimilarities between them.

13. Can we build algebraic codes on neutrosophic triplet groups using max min or max-product operations?.

14. Let S = $\{Z_{14}, \times\}$ be the semigroup under product. The neutrals of S are 7 and 8.

The neutrosophic triplets groups are $P_1 = \{(8, 8, 8), (2, 8, 4), (4, 8, 2), (6, 8, 6), (12, 8, 10), (10, 8, 12)\}$ and $P_2 = \{(7, 7, 7), (0, 0, 0)\}$.

Now we build codes using max product rule using $P_1 \cup \{(0, 0, 0)\}$.

Let G =

$$\begin{bmatrix} (2,4,8) & (0,0,0) & (10,8,12) & (0,0,0) & (8,8,8) \\ (0,0,0) & (12,8,10) & (0,0,0) & (2,8,4) & (0,0,0) \\ (6,8,6) & (2,8,4) & (0,0,0) & (0,0,0) & (4,8,2) \end{bmatrix}_{3\times 5}$$

3×5 generator matrix of a (5, 3) linear code with entries from the set P_1.

The code words are those that take its values from P_1.

Let x = ((6, 8, 7), (0, 0, 0), (2, 8, 4)) be the input neutrosophic triplet vector.

We find max product

{x, G} = ((12, 8, 10), (4, 8, 2), (4,8, 2), (0, 0, 0), (6, 8, 6)), is the generated code word.

Likewise can we find the code collection?

Note: This part is dealt in the problem session as we have not yet constructed matrices with entries using the neutrosophic triplet groups.

The advantages etc. of using this in terms of error correction etc; can be dealt as a research problem.

However the main advantage of the triple is the middle term is always fixed but if one of the end terms is wrong it can be corrected however there is at times ambiguity.

15. Construct using the classical group of neutrosophic triplet groups built using $\{Z_{46}, \times\}$ associated with the neutral

element $24 \in Z_{46}$, a (7, 4) code and use the max product (or max min) operation and develop all classical properties of this code.

a) How is this code different from the classical one?

b) What are advantages and disadvantages of using these codes?

c) What are the probable applications of these codes?

16. Let $S = \{Z_{74}, \times\}$ be the semigroup under product modulo 74.

    a) Find the neutrosophic triplet groups collection W using $38 \in Z_{74}$ as the neutral element.

    b) Construct a (7, 4) code using W under (i) max product, ii) max min.

    c) What are the main advantages of using these codes?

Chapter Three

SPECIAL TYPE OF NEUTROSOPHIC TRIPLET GROUPS MATRICES

We have analysed the properties of neutrosophic triplet groups built over Z_{2p}, Z_{pq} and Z_n where $n = 2^2p$ or 3^2p, p a prime different from 2 and 3.

We saw in several cases the collection of neutrosophic triplet groups formed a group, sometimes cyclic under product modulo n. For special cases of Z_n, n not of any form mentioned above formed a semigroup under product modulo n.

We also defined two special notions, quasi neutrosophic triplet groups and duplets associated with the idempotents or the neutral elements of Z_n.

In this chapter we proceed onto define the new notion of neutrosophic triplet group matrices under natural product $\times_n$ modulo n. At the outset we wish to record many a times the collection of neutrosophic triplet groups in general may not be

even compatible under modulo addition. Keeping all this in view we define only on matrices natural products $\times_n$

However in case of adopting these neutrosophic triplet groups matrices to mathematical models we can define the notion of max-product and max-min using them. All these concepts will be developed, described and defined in this chapter. The next chapter we will develop the possible applications of these newly built models.

Before we make any abstract definition of these new concepts we describe them by appropriate examples in order to make it easy for the reader.

For the structure of neutrosophic triplet groups classical groups and the very construction of them the reader is requested to refer the earlier chapter where these notions are dealt elaborately.

Example 3.1. Let S = $\{Z_{26}, \times\}$ be the semigroup under product modulo 26. The idempotents or neutral elements of Z_{26} are 13 and 14. For 26 = 2.13 then the idempotents are 13 and 13 + 1.

We now find all the neutrosophic triplet groups of Z_{26} related to the neutral element 14.

(2, 14, 20) and (20, 14, 2) are neutrosophic triplet groups.

(4, 14, 10) and (10, 14, 4) are neutrosophic triplet groups.

(8, 14, 18) and (18, 14, 8) are neutrosophic triplet groups.

(16, 14, 22) and (22, 14, 16) are neutrosophic triplet groups.

(6, 14, 24) and (24, 14, 6) are neutrosophic triplet groups.

(12, 14, 12) and (14, 14, 14) are neutrosophic triplet groups.

Clearly if A = {(2, 14, 20), (20, 14, 2), (4, 14, 10), (10, 14, 4), (8, 14, 18), (18, 14, 8), (16, 14, 22), (22, 14, 16), (6, 14, 24), (24, 14, 6), (12, 14, 12), (14, 14, 14)} is the classical group of neutrosophic triplet groups with (14, 14, 14) as the identity element under product modulo 26.

Infact A is a cyclic group of order 12 generated by (2, 14, 20) as it is easily verified

$$(2, 14, 20)^{12} = \underbrace{(2,14,20) \times \ldots \times (2,14,20)}_{12 \text{ times}} = (14, 14, 14).$$

Now if

x = ((20, 14, 2), (4, 14, 10), (12, 14, 12)) be a 1 × 3 row matrix with entries from A now we perform operations on them.

x × x = (20, 14, 2), (4, 14, 10), (12, 14, 12)) × ((20, 14, 2), (4, 14, 10), (12, 14, 12)) = ((10,14, 4), (16, 14, 22), (14, 14, 14)).

We can find $x^2 \times x = x^3$ = ((10, 14, 4), (16, 14 22), (14, 14, 14)) × ((20, 14, 2), (4, 14, 10), (12, 14, 12)) = ((18, 14, 8), (12, 14, 12), (12, 14, 12)) and so on.

However we see this will yield the row matrix identity for some n as

x^n = ((14, 14, 14), (14, 14, 14), (14,14, 14)), n > 3 for this case and so on.

Let x = ((6, 14, 24), (22, 14, 16), (14, 14, 14)) and

y = ((4, 14, 10), (8, 14, 18), (2, 14, 20)) be two row matrices of neutrosophic triplet groups.

We find x × y, x × y = ((6, 14, 24), (22, 14, 16), (14, 14, 14)) × ((14, 14, 10), (8, 14, 18), (2, 14, 20) ={(24, 14, 6), (14, 14, 14), (2, 14, 20))}.

It is easily verified that product of two row matrices is both commutative and associative.

Let B = {collection of all 1 × 3 row matrices with entries from A} = {(a, b, c) / a, b, c ∈ A} be the row matrix of neutrosophic triplet groups.

Clearly {B, ×} is a group under product of finite order.

Infact o(B) = 12^3.

If x = ((18, 14, 8), (24, 14, 6), (12, 14, 12)) ∈ B then inverse of x in B under product modulo 26 is y = {(8, 14, 18), (6, 14, 24), (12, 14 12)} in B.

It is easily verified y is unique and x × y = y × x = ((14, 14, 14), (14, 14, 14)) and this element ((14, 14, 14), (14, 14, 14), (14, 14, 14)) in B acts as the 1 × 3 row matrix identity.

The following facts about B are important.

In the first place B is a group only under product modulo 26.

Infact B is not even closed under the sum.

For if x = ((2, 14, 20), (8, 14, 18), (6, 14, 24)) $\in$ B then x + x = ((2, 14, 20), (8, 14, 18), (6, 14, 24)) + ((2, 14, 20), (8, 14, 18), (6, 14, 4)) = ((4, 2, 14), (16, 2, 10), (12, 2, 22)) we see none of the entries in x + x is an element of A so cannot be an element of B.

Thus the sum operation cannot be defined on B.

Now we consider

C = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} / a_i \in A; 1 \leq i \leq 4\right\}$ to be the collection of all 4 $\times$ 1 column neutrosophic triplet group matrices.

If we define natural product $\times_n$ on C then we see $\{C, \times_n\}$ is again a group of neutrosophic triplet groups of order 12^4.

We will just indicate how the product operation $\times_n$ is performed on C.

Let x = $\begin{bmatrix} (6,14,24) \\ (20,14,2) \\ (8,14,18) \\ (12,14,12) \end{bmatrix}$ and y = $\begin{bmatrix} (20,14,2) \\ (24,14,6) \\ (10,14,4) \\ (8,14,18) \end{bmatrix}$ be in C.

We find the natural product $\times_n$.

$$x \times_n y = \begin{bmatrix} (6,14,24) \\ (20,14,2) \\ (8,14,18) \\ (12,14,12) \end{bmatrix} \times_n \begin{bmatrix} (20,14,2) \\ (24,14,6) \\ (10,14,4) \\ (8,14,18) \end{bmatrix} = \begin{bmatrix} (16,14,22) \\ (12,14,12) \\ (2,14,20) \\ (18,14,8) \end{bmatrix} \in C.$$

It is easily verified the product $\times_n$ on C is both commutative and associative. Closure exist as A is a group under product modulo 26.

Now we find inverse of any x in C. We first claim for every x in C we have a unique y in C such that

$$x \times_n y = \begin{bmatrix} (14,14,14) \\ (14,14,14) \\ (14,14,14) \\ (14,14,14) \end{bmatrix} \text{ the identity matrix of the collection C.}$$

Let $x = \begin{bmatrix} (4,14,10) \\ (24,14,6) \\ (16,14,2) \\ (8,14,18) \end{bmatrix}$ of y in C is as follows.

$y = \begin{bmatrix} (10,14,4) \\ (6,14,24) \\ (22,14,16) \\ (18,14,8) \end{bmatrix} \in$ C is such that

$$x \times_n y = \begin{bmatrix} (4,14,10) \\ (24,4,6) \\ (16,14,22) \\ (8,14,18) \end{bmatrix} \times_n \begin{bmatrix} (10,14,4) \\ (6,14,24) \\ (22,14,16) \\ (18,14,8) \end{bmatrix} = \begin{bmatrix} (14,14,14) \\ (14,14,14) \\ (14,14,14) \\ (14,14,14) \end{bmatrix}$$ which

identity matrix of C.

Infact order C is 12^4 and every element x in C is of finite order.

Next we consider

$$D = \{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} / a_i \in A, 1 \leq i \leq 9\}$$

to be the collection of all 3 × 3 matrices of neutrosophic triplet groups.

We find the natural product operation $\times_n$ on D. However the usual or the classical product cannot be defined on D.

This will also be established in due course of the discussions.

Let $$x = \begin{bmatrix} (2,14,20) & (8,14,18) & (12,14,12) \\ (14,14,14) & (6,14,24) & (18,14,8) \\ (4,14,10) & (14,14,14) & (20,14,20) \end{bmatrix}$$ and

$$y = \begin{bmatrix} (14,14,14) & (12,14,12) & (16,14,22) \\ (2,14,20) & (14,14,14) & (20,14,2) \\ (4,14,10) & (16,14,4) & (12,14,12) \end{bmatrix}$$ in D.

We first find $x \times_n y$.

$$x \times_n y = \begin{bmatrix} (2,14,20) & (2,14,20) & (10,14,4) \\ (2,14,20) & (6,14,24) & (22,14,16) \\ (16,14,22) & (10,14,4) & (6,14,24) \end{bmatrix} \in D.$$

This is the way the natural product $\times_n$ is performed on D.

Now we show the classical product cannot be defined on D.

$$\text{Let } x = \begin{bmatrix} (2,14,20) & (20,14,2) & (14,14,14) \\ (10,14,4) & (12,14,12) & (4,14,10) \\ (6,4,24) & (8,14,18) & (12,14,12) \end{bmatrix} \text{ and}$$

$$y = \begin{bmatrix} (4,14,10) & (12,14,12) & (8,14,8) \\ (14,14,14) & (18,14,8) & (2,14,20) \\ (12,14,12) & (14,14,4) & (4,14,10) \end{bmatrix} \in D$$

We find $x \times y$ where '$\times$' is the usual product (or classical product) of x with y.

$$x \times y = \begin{bmatrix} (2,14,20) & (20,14,2) & (14,14,14) \\ (10,14,4) & (12,14,12) & (4,14,10) \\ (6,4,24) & (8,14,18) & (12,14,24) \end{bmatrix} \times$$

$$\begin{bmatrix} (4,14,10) & (12,14,12) & (8,14,18) \\ (14,14,14) & (18,14,8) & (2,14,20) \\ (12,14,12) & (14,14,14) & (4,14,10) \end{bmatrix} =$$

$$\begin{bmatrix} (14,16,6) & (8,16,10) & (10,16,20) \\ (22,16,16) & (12,16,14) & (16,16,22) \\ (20,16,12) & (16,16,22) & (8,16,2) \end{bmatrix} \notin D;$$

further it is pertinent to note that none of the entries in $x \times y$ is from A.

That is why we cannot define the classical product operation on D. In fact addition modulo 26 is not defined in A or in particular A is not closed under the operation + modulo 26 on D.

It is left for the reader to verify that the operation natural product $\times_n$ modulo 26 is both associative and commutative.

In fact $\{D, \times_n\}$ is a group of order 12^9 and is defined as the classical group matrix of neutrosophic triplet groups.

$$\text{Further } I = \begin{bmatrix} (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) \end{bmatrix} \text{ in } D$$

acts as the multiplicative identity of D, the classical group of neutrosophic triplet group.

We see every $x \in D$ is of finite order in fact $x^n = I$ for some $n \geq 2$ such that $n / 12^9$.

Suppose we take

E = {collection of all 4×3 matrices with entries from A}

$$= \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \Big/ a_i \in A; 1 \leq i \leq 12 \right\} \text{ be the collection of all}$$

4×3 neutrosophic triplet group matrices with entries from A.

We can define the natural product $\times_n$ modulo 26 on E.

We just show how product $\times_n$ is defined on E.

Let x = $\begin{bmatrix} (14,14,14) & (2,14,20) & (8,14,8) \\ (20,14,2) & (18,14,8 & (12,14,12) \\ (4,14,10) & (14,14,14) & (10,14,4) \\ (6,14,24) & (2,14,20) & (8,14,18) \end{bmatrix}$ and

y = $\begin{bmatrix} (12,14,12) & (14,14,14) & (6,14,24) \\ (6,14,24) & (20,14,2) & (2,14,20) \\ (24,14,6) & (18,14,8) & (4,14,10) \\ (10,14,4) & (14,14,14) & (12,14,12) \end{bmatrix} \in$ E.

We find out $x \times_n y$.

$$x \times_n y = \begin{bmatrix} (12,14,12) & (2,14,20) & (22,14,16) \\ (16,14,22) & (22,14,16) & (24,14,6 \\ (18,14,8) & (18,14,8) & (14,14,14) \\ (8,14,18) & (2,14,20) & (18,14,8) \end{bmatrix}.$$

This is the way product operation $\times_n$ is performed on E. Clearly E is a commutative group of order 12^{12} with

I = $\begin{bmatrix} (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) \end{bmatrix}$ as the identity with respect to the natural product $\times_n$.

Let F = {collection of all 2 × 5 matrices with entries from A}

= $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix}\right.$ / $a_i \in A$; $1 \leq i \leq 10$} be the collection of all neutrosophic triplet group matrices.

Clearly {F, $\times_n$} is an abelian group of order 12^{10} with

I = $\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \end{bmatrix}$ / $a_i \in A$, $1 \leq i \leq 10$}

be the collection of all neutrosophic triplet group matrices.

Clearly {F, $\times_n$} is an abelian group of order 12^{10} with

I =

$$\begin{bmatrix} (14,14,14) & (14,14,14) & (14,14,14) & (14,14,14) & (14,14,14) \\ (14,14,14) & (14,14,14) & (14,14,14) & (14,14,14) & (14,14,14) \end{bmatrix}$$

as its multiplicative identity under the natural product $\times_n$.

Now we make the abstract definition of the classical group of neutrosophic triplets group under the natural product $\times_n$.

Definition 3.1. *Let S = {Z_n, ×} where n = 2p, p an odd prime.*

Let A = {collection of all neutrosophic triplet groups}. {A, ×} is a classical cyclic group of neutrosophic triplet groups of order (p – 1).

B = {collection of all s × t matrices with entries from A} B is defined as the classical group of neutrosophic triplet groups under natural product $\times_n$ of order $(s \times t)^{p-1}$.

We have given an example of it.

Now we proceed onto give an example in which

$n = 2^2 . 3 = 12$.

Example 3.2. Let S ={Z_{12}, ×} be the semigroup under product modulo 12. The idempotents (neutral elements of Z_{12}) are 4 and 9.

The neutrosophic triplet groups are as follows.

(3, 9, 3) and (9, 9, 9) are neutrosophic triplet groups.

(8, 4, 8) and (4, 4, 4) are neutrosophic triplet.

We see $2 \times 9 \equiv 6 \pmod{12}$.

$2 \times 4 \equiv 8 \pmod{12}$ so 2 cannot contribute to any neutrosophic triplet groups.

$6 \times 9 = 6 \pmod{12}$ and $6 \times 4 \equiv 0 \pmod{12}$.

We see the neutral of 9 is six but there is no anti 6 which is such that anti $6 \times 9 = 9 \pmod{12}$.

$10 \times 9 = 6 \pmod{12}$ and $10 \times 4 \equiv 4 \pmod{12}$.

Thus (10, 4, 10) is a quasi neutrosophic group as

$4 \times 10 \equiv 4 \pmod{12}$ and $10 \times 10 \equiv 4 \pmod{12}$.

(6, 9) is a neutrosophic duplet (0, 9) is again a neutrosophic duplet.

Here it is important to note that we do not exhaust all even elements of Z_{12} to contribute to neutrosophic triplet groups.

We see $o(Z_{12}) = 12$ but only two elements

A = {(3, 9, 3), (9, 9, 9)} are neutrosophic triplet groups associated with the neutral element 9.

{(4, 4, 4) and (8, 4, 8)} = B are neutrosophic triplet groups associated with the neutral element 4.

Further $A \times B = \{(0, 0, 0)\}$, A and B are classical cyclic groups of order two.

There is only one quasi neutrosophic triplet group which is non trivial.

The element $2 \in Z_{12}$ is not associated with any form of neutral element set.

Even though 12 is even still the deviant behavior when compared with 2p, p an odd prime is very striking.

We give one more example to analyse the above situation.

Example 3.3. Let $S = \{Z_{20}, \times\}$ be the semigroup under product modulo 20.

The neutral elements (idempotents) of Z_{20} are 5 and 16 neutrosophic triplet groups associated with the neutral element 16 are {(4, 16, 4), (8, 16, 12), (12, 16, 8), (16, 16, 16)}.

The classical group of neutrosophic triplet groups associated with the neutral element 5 is empty associated with 5 are the group of quasi neutrosophic triplet groups {(15, 5, 15), (5, 5, 5)}.

The special semigroup of duplets associated with 16 is as follows;

{(6, 16), (10, 16), (0, 16)}

The elements 10, 14 and 18 do not contribute to any of the special elements as it is clearly observed.

$10 \times 16 \equiv 0 \pmod{20}$, $10 \times 5 = 10 \pmod{20}$.

But these producted with any even elements leads to zero as only even numbers and multiplies of 5 can contribute to these special elements.

$14 \times 5 = 10 \pmod{20}$ and $14 \times 16 = 4 \pmod{20}$.

So 14 does not contribute to any special elements.

$18 \times 5 = 10 \pmod{20}$, $18 \times 16 = 8 \pmod{20}$.

So 18 also does not contribute to any type of special elements. The set {18, 14} does not yield any special elements.

Only {6, 10} yields the duplets with 16.

Hence it is at this juncture we have to keep on record that Z_{20} and Z_{12} may not yield neutrosophic triplet groups as given by Z_{26} or Z_{14} or Z_{10}.

Thus for the sake of completeness we describe the neutrosophic triplet groups associated with Z_{14} and Z_{10}.

Now the neutral elements associated with Z_{14} are 7 and 8 respectively.

Likewise the neutral elements of Z_{10} are 5 and 6.

Finally the neutral elements of Z_{2p} (p an odd prime) are p and p + 1.

In fact all the elements of Z_{14} (or Z_{10}) which are not units contribute to neutrosophic triplet groups only associated with the 8 (or 6) not with 7 (or 5).

The order of the set of all neutrosophic triplet groups of Z_{14} is 6. {(2, 8, 4), (4, 8, 2), (6, 8, 6), (10, 8, 12), (12, 8, 10) and (8, 8, 8)} are neutrosophic triplets of Z_{14} associated with 8.

Similarly {(2, 6, 8), (8, 6, 2), (6, 6, 6), (4, 6, 4)} are neutrosophic triplet groups associated with Z_{10}.

Hence it is difficult get for all n, $n \neq 2p$, p an odd prime.

It is kept on record that for $n = 2^2p$, p any odd prime we do not have many elements which contribute to neutrosophic triplet groups.

Let us consider Z_{15} where 15 = 3.5 we find the classical group matrix of neutrosophic triplet groups of Z_{15}.

The idempotents of Z_{15} are 6 and 10. The neutrosophic triplet groups collection associated with the neutral element 6 is {(3, 6, 12), (12, 6, 3), (4, 6, 9), (6, 6, 6)} and the neutrosophic triplet groups associated with the neutral element 10 is {(5, 10, 5), (10, 10, 10)}.

It is interesting to record that if in Z_n, n = 3 × p, p an odd prime different from zero that we can say there will be only two distinct neutrosophic triplet groups associated with the two neutrals and order of one of them will be p – 1 and that of the order only two.

We consider the following examples.

Example 3.4. Let S = {Z_{21}, ×} be the semigroup under product modulo 21. 21 = 3.7 is of the form 3p. The neutral elements of S are 7 and 15.

The neutrosophic triplet groups associated with 7 are {(7, 7, 7) and (14, 7, 14)} is a group of order two.

The neutrosophic triplet groups associated with 15 are {(3, 15, 12), (12, 15, 3), (15, 15, 15), (9, 15, 18), (18, 15, 9), (6, 15, 6)}.

Clearly the number of such triplets are 6 in keeping without prediction.

Consider another example.

Example 3.5. Let S = {Z_{69}, ×} be the semigroup under product modulo 69.

The neutral elements of S are 24 and 46.

The neutrosophic triplet groups associated with 46 are {(46, 46, 46), (23, 46, 23)}. Clearly order of the group is two.

The neutrosophic triplet groups associated with 24 are;

B = {(3, 24, 54), (54, 24, 3), (9, 24, 18), (18, 24, 9), (27, 24, 6), (6, 24, 27), (12, 24, 48), (48, 24, 12), (36, 24, 39), (39, 24, 36), (21, 24,57), (57, 24, 21), (30, 34, 33), (33, 24, 30), (66, 24, 15), (15, 25, 6), (24, 24, 24), (60, 24, 51), (51, 24, 60), (45, 24, 45), (63, 24, 42), (42, 24, 63)}.

Clearly there are 22 neutrosophic triplet groups so o(B) = 22 = 23 – 1. Thus we once again mention that it is a open problem to prove if in Z_n, n = 3p, p an odd prime different from 3 then for the semigroup $\{Z_n, \times\}$ under product modulo n we have the following.

i) Prove only two neutrals (idempotents) in Z_{3p}.

ii) Prove we have only one of the neutrals which contribute to (p – 1) neutrosophic triplet groups which forms a group under product modulo 3p.

iii) Prove there is only one cyclic group of order two.

iv) Will the collection of elements in Z_n which contribute for the neutrosophic triplet groups is of order p and one of them is a pseudo primitive element of that collection.

Consider yet another example.

Example 3.6. Let S = $\{Z_{91}, \times\}$ the semigroup under product modulo.

The reader is left with the task of finding neutral elements and the related neutrosophic triplet groups.

Now having defined matrices of all types using neutrosophic triplet groups we now proceed on to describe and develop them for mathematical models.

Before we proceed onto define mathematical models we describe with some more examples the concept of neutrosophic triplet groups matrix collection in the following.

Example 3.7. Let S = $\{Z_{51}, \times\}$ be the semigroup under product modulo 51.

The neutral elements of Z_{51} are 34, 18 $\in Z_{51}$ is such that $18 \times 18 \equiv$ (mod 51) and $34 \times 34 \equiv 34$ (mod 51).

We now find the neutrosophic triplet group sets associated with 18 and 34, the neutral elements of Z_{51} under product modulo 51.

The neutrosophic triplet groups associated with the neutral element 18 are

A = {(3, 18, 6), (6, 18, 3), (9, 18, 36), (36, 18, 9), (18, 18, 18), (33, 118, 33), (27, 18, 12), (12, 18, 27), (15, 18, 42), (42, 18, 15), (30, 18, 21), (21, 18, 30), (39, 18,24), (24, 18, 39), (45, 18, 48), (48, 18, 45)}.

The neutrosophic triplet groups associated with 34 are {(17, 34, 17), (34, 34, 34)} = B

both A and B are cyclic groups of order 16 and 2 respectively.

We can build matrices using A and perform some more operations on them.

$$\text{Let } A = \begin{bmatrix} (3,18,6) & (18,18,18) & (42,18,15) \\ (9,18,36) & (15,18,42) & (18,18,18) \\ (18,18,18) & (33,8,33) & (3,18,6) \\ (6,18,3) & (18,18,18) & (35,18,33) \\ (12,18,27) & (15,18,42) & (42,18,15) \end{bmatrix}$$

be a 5×3 neutrosophic triplet groups matrix.

We know we can define natural product $\times_n$ operation on the collection of all 5×3 matrices.

Now we wish to define other types of operations on them.

So in the first place we define operations on the set A and then we can easily transform them to the matrices of neutrosophic triplet groups.

It is pertinent in the first place to keep on record A is not even closed under the modulo addition +.

For if x = (6, 18, 3) and y = (33, 18, 33) are in A, then x + y = (6, 18, 3) + (33, 18, 33) = (39, 36, 36) $\notin$ A so addition '+' modulo 51 is not defined on A.

However A under product modulo 51 is a cyclic group of order 16.

Let x = (33, 18, 33) and y = (6, 18, 3) $\in$ A.

x $\times$ y = (33, 18, 33) $\times$ (6, 18, 3) = (198, 324, 99) taking mod 51, x $\times$ y = (45, 18, 48) $\in$ A.

Infact we can prove A under product modulo 51 is a cyclic group of order 16.

Now we proceed onto define max operation on A.

Let x = (39, 18, 24) and y = (15,18,42) $\in$ A.

max {x, y} = max {(39, 18, 24), (15, 18, 42)} = {(39, 18,42)} $\notin$ A so is not even closed under the max operation.

Consider min operation on A, min {x, y} = min {(39, 18, 24), (15, 18, 42)} = (15, 18, 24) $\notin$ A neither can we define min operation on A.

So only the operation $\times$ on A can be extended the operation of natural product on matrices.

So on the class of all neutrosophic triplet group matrices we cannot define the operations + or max or min as on A we are not in a position to define these operations.

Only with these limitations we have to work.

We further give other types of $\times$ operation on the collection of all neutrosophic triplet groups matrices.

Let M = {(collection of all 2 $\times$ 4 matrices with entries from A = {(18, 18, 18), (6, 18, 3), (3, 18, 6), (9, 18, 36), (36, 18, 9), (33, 18, 33), (27, 18, 12), (12, 18, 27), (15, 18, 42), (42, 18, 15), (48, 18, 45), (45, 18, 48), (30, 18, 21), (21, 18, 30), (39, 18, 24), (24, 18, 39)}} be the collection of all 2 $\times$ 4 matrices.

If M $\in$ A and x = (9, 18, 36) $\in$ A. We x $\times$ M where

$$M = \begin{bmatrix} (36,18,9), & (15,18,42) & (48,18,45) & (24,18,39) \\ (33,18,33) & (18,18,18) & (30,18,21) & (27,18,12) \end{bmatrix}.$$

$$(9, 18, 36) \times \begin{bmatrix} (36,18,9), & (15,18,42) & (48,18,45) & (24,18,39) \\ (33,18,33) & (18,18,18) & (30,18,21) & (27,18,12) \end{bmatrix}$$

$$= \begin{bmatrix} (18,18,18) & (33,18,33) & (24,18,39) & (12,18,27) \\ (42,18,15) & (9,18,36) & (15,18,42) & (39,18,24) \end{bmatrix} \in M.$$

This is the way product operation is performed.

As we are not in a position to define min or max operation and without these two operations on M or for that matter on any collection of neutrosophic triplet group matrices we are forced to define the new notion of conditionally defined neutral property max c.n function conditionally defined neutral property min on neutrosophic triplet group matrices. So we first define the notion of conditionally defined neutral property min (c.n min) on A.

Let x = (12, 18, 27) min and y = (42, 18, 15) $\in$ A conditionally defined neutral min property (c.n min) of {x, y} denoted by c.n - min {x, y}} = c.n min {(12, 18, 27), (42, 18,15)} = {(min {12, 42}, min {18, 18}, anti of min {12, 42})} = (12, 18, 27).

It is pertinent to note actually min {27, 15} = 15 and (12, 18, 15) does not form a neutrosophic group triplet, that is (12, 18, 15) $\notin$ A.

Next we proceed onto define the notion of conditionally defined neutral max (c.n max) on A as follows.

Let x = (12, 18, 27) and y = (42, 18,15)

c.n max {(12,18, 27), (42, 18, 15)} = (max {12, 42}, max {18, 18}, anti max ({12, 42}) = (42, 18, 15).

That is in case of conditionally neutral max we only find max of {a, b} say it is a then associate with it the anti max {a,b} so c.n. max{anti a, anti b} = anti max {a, b}.

We will put them in the following way let x = (a neu a, anti a) and y = (b, neut a, anti b) $\in$ A.

c.n max { x,y } = {(max {a, b}, neut a, anti max {a, b})

Thus if x = (27, 18, 12) and y = (30, 18, 21).

Then c.n. max{x, y} = c.n max {(27, 18, 12), (30, 18, 21), = (max (27, 30), 18, anti max {27, 30}) = (30, 18, anti 30) = (30, 18, 21) $\in$ A.

On similar lines we define conditionally neutral min of two neutrosophic triplet groups.

Let x = (27, 18, 12) and y = (30, 18, 21) $\in$ A

min {x, y} = min {(27, 18, 12),(30, 18, 21)} = (min {27, 30}, 18, anti min {27, 30}) = (27, 18 anti 27) = (27, 18, 12).

Now we find c - n - max of x = (9, 18, 36) and y = (30, 18, 21) in A.

c.n max {x, y} = c. n. max {(9, 18, 36), (30, 18, 21)} = (max {9, 30}, 18, anti max {9, 30}) = (30, 18, anti 30) =(30, 18, 21) $\in$ A.

It is interesting to observe that max {36, 21} is only 36 and (30, 18, 36) is not a neutrosophic triplet group with 18 as the neutral element.

We find c.n min {x, y} = c.n min {(9, 18, 36), (30, 18, 21)} = (min{9, 30}, 18, anti min {9, 30}) = (9, 18, anti 9) = (9, 18, 36). However it is clearly observed min {36, 21} = 21 and (9, 18, 21) does not form a neutrosophic triple group of the neutral element 18.

Thus we see A under the operation c.n min is a semigroup. That is {A, c.n min} is a semigroup of order 16 or equivalently A is closed under c.n-min and {A, c.n. min} is a semigroup which is commutative. However {A, c.n min} is not a monoid.

Now let us consider (9, 18, 36) and (30, 18, 21) in A.

c.n max {(9, 18, 36), (30, 18, 21)} = (max {9, 30}, 18 antimax {9, 30}) = (30, 18, anti 30) = (30, 18, 21) $\in$ A.

So {A, c.n - max} is a semigroup which is not a monoid.

Only by defining these two special type of operations on A we can define successfully a semigroup.

We will for the sake of better understanding give an example.

Example 3.8. Let S = $\{Z_{33}, \times\}$ to be a semigroup under product modulo 33. The neutral elements of S are 12 and 22. The neutrosophic triplet groups associated with 12 the neutral element are

A = {(3, 12, 15), (15, 12, 3), (9, 12, 27), (12, 12, 12), (27, 12, 9), (6, 12, 24), (24, 12, 6), (18, 12, 30), (30, 12, 18), (21, 12, 21)}.

The neutrosophic triplet group associated with 22 are {(22, 22, 22), (11, 22, 11)}.

Clearly {A, $\times$} is a cyclic group of order 9.

{A, c.n max} and {A, c.n min} are semigroups which are not monoids.

We now show how c.n max and c.n min operations are performed on matrices of neutrosophic triplet groups.

Let {3 $\times$ 5 matrices with entries from A} =

$$\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \middle/ a_i \in A; 1 \leq i \leq 15\right\}$$

be the collection of all matrix of neutrosophic triplet groups.

We show {M, c.n - min} is a semigroup and is not a monoid.

Let x =

$$\begin{bmatrix} (3,12,15) & (30,12,18) & (21,12,21) & (6,12,24) & (12,12,12) \\ (12,12,12) & (18,12,30) & (24,12,6) & (12,12,12) & (24,12,6) \\ (3,12,15) & (15,12,3) & (12,12,12) & (18,12,30) & (15,12,3) \end{bmatrix}$$

$\in$ A.

If α = (27, 12, 9) $\in$ A to find

c.n - min {α,x } =

$$\begin{bmatrix} (3,12,15) & (27,12,9) & (21,12,21) & (6,12,24) & (12,12,12) \\ (12,12,12) & (18,12,30) & (24,12,6) & (12,12,12) & (24,12,6) \\ (3,12,15) & (15,12,3) & (12,12,12) & (18,12,30) & (15,12,3) \end{bmatrix}.$$

Clearly c.n min {x, x} = x for all x $\in$ A.

Infact {M, c.n min} is a semigroup under c.n - min operation. However M has no identity.

Let T = {Collection of all 5 $\times$ 2 matrices with entries

from A} = $\left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \right.$ / $a_i \in$ A; $1 \leq i \leq 10$} = collection of all

5 $\times$ 2 matrices with entries from A.

We now show how a special type of c.n - max operation is performed on T using elements from A.

Let x = (3, 12, 15) $\in$ A and

$$a = \begin{bmatrix} (12,12,12) & (18,12,30) \\ (15,12,3) & (30,12,18) \\ (21,12,21) & (6,12,24) \\ (9,12,27) & (24,12,6) \\ (18,12,30) & (6,12,24) \end{bmatrix} \in T.$$

We define c.n - max {x, a} = $\begin{bmatrix} (12,12,12) & (18,12,30) \\ (15,12,3) & (30,12,18) \\ (21,12,21) & (6,12,24) \\ (9,12,27) & (24,12,6) \\ (18,12,30) & (6,12,24) \end{bmatrix}$ = a $\in$ T.

In this way {T, c.n max} happens to be only a semigroup and not a monoid.

Let W = {collection of all 4 $\times$ 4 matrices with entries from A}

$$= \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right.$$

where $a_i \in$ A, $1 \leq i \leq 16$} = collection of all 4 $\times$ 4 matrices with entries from A. Let x = (6, 12, 24) $\in$ A and

$$\alpha = \begin{bmatrix} (24,12,6) & (12,12,12) & (3,12,15) & (24,12,6) \\ (15,12,3) & (18,12,30) & (27,12,9) & (9,12,27) \\ (21,12,21) & (12,12,12) & (24,12,6) & (9,12,27) \\ (12,12,12) & (9,12,27) & (6,12,24) & (15,12,3) \end{bmatrix}$$ be in W.

We find x $\times$ α = (6, 12, 24) $\times$ α =

$$\begin{bmatrix} (12,12,12) & (6,12,24) & (18,12,30) & (12,12,12) \\ (24,12,6) & (9,12,27) & (30,12,18) & (24,12,6) \\ (27,12,9) & (6,12,24) & (12,12,12) & (21,12,21) \\ (6,12,24) & (21,12,21) & (3,12,15) & (24,12,6) \end{bmatrix} \in T.$$

This is the way special type of product operation is performed from $\alpha \in T$ and $x \in A$.

Now we perform row matrix into a matrix by defining either max product or max min operations.

Let x = ((3, 12, 15), (12,1 2,12), (15, 12, 3), (30, 12, 18), (21, 12, 21)) be a 1×5 matrix.

$$\text{Let } a = \begin{bmatrix} (6,12,24) & (24,12,6) \\ (12,12,12) & (6,12,24) \\ (24,12,6) & (15,12,3) \\ (3,12,15) & (21,12,21) \\ (21,12,21) & (6,12,24) \end{bmatrix} \in T$$

max min {x, a} = ((21, 12, 21),(21, 12, 21)).

This is the way max min operation is performed.

max - product {x, a} = ((30, 12, 18), (27, 12, 9)).

We see max product operation is defined in this way.

It is pertinent to keep on record that always max product will yield a bigger value than max min.

Now this sort of product will be used in mathematical models which will be constructed in the next chapter.

Now we wish to show that these sort of operation will after a stage end in a fixed point. To this end we will construct another example using both square neutrosophic triplet group matrices and rectangular neutrosophic triplet group matrices.

Example 3.9. Let S = $\{Z_{21}, \times\}$ be the semigroup under product modulo 21, 21 = 3.7. The neutral elements of S are 7 and 15.

The neutrosophic triplet groups associated with 15 are

A = {(15, 15, 15), (3, 15, 12), (9, 15, 18), (12, 15, 3), (18, 15, 9), (6, 15, 6)}.

Now we show if M = {collection of all 2 × 2 matrices with entries from A}

$= \left\{\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} / a_i \in A; 1 \leq i \leq 4\right\}$ = Collection of all

2 × 2 neutrosophic triplet group matrices with entries from A.

Let X = $\{(a_1, a_2) / a_i \in A; 1 \leq i \leq 2\}$ be the collection of all row matrices with entries from A.

Now if $x \in A$ and $N \in M$ then if $xN = y_1$ find $y_1N = y_2$ say then we claim after a finite number of such iterations we get $y_t^N = y_{t+1}$ with $y_{t+1}N = y_{t+1}$ and so on will yield a fixed point or $y_1 \to y_2 \to \ldots \to y_{t+1} \to \ldots \to y_{t+1}$ a limit cycle. This concept will be used in the mathematical models.

We will just illustrate this by some examples.

Let x = ((3, 15, 12), (6, 15, 6)) $\in$ X and

$$N = \begin{pmatrix} (12,15,3) & (18,15,9) \\ (9,15,18) & (3,15,12) \end{pmatrix} \in M;$$

we find c.n max-product {x, N};

c.n max-product {x, N} = c.n max-product {((3, 15, 12), (6, 15, 6)), N} = ((15, 15, 15), (18, 15, 9)) = y_1 say,

c-n max-product {y_1, N} = ((15, 15, 15), (18, 15, 9)) =

y_2 (say) = y_1 I

Thus we see at the second iteration itself the term x on N with c.n max product converges to ((15, 15, 15), (18, 15, 9)) or in technical terms yields a fixed point.

Now we find for the same x the c.n max min {(x, N)}.

c.n max-min {(x, N)} = ((6, 15, 6), (3, 15, 12)) = z_1 (say); now c.n maxmin {(z_1, N)} = ((6, 15, 6), (6, 15, 6)) = z_2 (say) c.n max min {z_2, N} = ((6, 15, 6), (6, 15, 6)) = z_3 (= z_2) II

Thus this row vector or row matrix of neutrosophic triplet groups converges to z_2 = ((6, 15, 6) (6, 15, 6)) or yield the fixed point z_2.

Hence it is pertinent to record at this juncture that the values or converging vector in case of c.n max-product (x, N) is different from the c.n max-min (x, N) which clearly evident from I and II.

Thus we can define in case of matrix of neutrosophic triplet groups the notion of special type of fixed points which is as follows.

Definition 3.2. *Let S = {Z_{pq}, ×} be the semigroup under product modulo pq (p and q two distinct primes). A = {Collection of all neutrosophic triplet groups associated with the neutral elements of Z_{pq}} of order p – 1 or (q – 1) depending on the neutral*

element of Z_{pq}. M = {collection of all $n \times n$ matrices with entries from A} be the collection of all $n \times n$ matrix of neutrosophic triplet groups.

Let X = {(a_1, …, a_n) / $a_i \in$ A, $1 \leq i \leq n$} be the collection of $1 \times n$ row vector or row matrices of neutrosophic triplet group.

Then c.n max-product (x, N) and cn max min (x, N) for every $x \in X$ and $N \in M$ either converges to a fixed point defined as the special fixed point or is a limit cycle.

We will illustrate this situation by one more example.

Example 3.10. Let S = {Z_{55}, ×} be the semigroup under product modulo 55.

The neutral elements of Z_{55} are 11 and 45.

We see the neutrosophic triplet groups associated with the neutral element 45 are as follows;

A ={(45, 45, 45), (5, 45, 20), (20, 45, 5), (25, 45, 15), (15, 45, 25), (10, 45, 10), (30, 45,40), (40, 45, 30), (35, 45, 50), (50, 45, 30)}.

We can for the neutral element 11 find the associated collection of all neutrosophic triplet groups.

Now using the collection A we construct the following matrix group of neutrosophic triplet groups.

$M = \{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}$ / $a_i \in A$, $1 \leq i \leq 16\}$ be the 4 $\times$ 4 matrix group of neutrosophic triplet groups.

$X = \{(a_1, a_2, a_3, a_4) / a_i \in A, 1 \leq i \leq 4\}$ be the collection of all row vector or row matrices of neutrosophic triplet groups.

For every $x \in X$ and $N \in M$ we find c.n max product {x, N} and c-n max min {x, N} and prove the limit converges to a fixed point or repeats itself as a limit cycle.

We will work out for a particular values of $x \in X$ and N $\in$ M.

Let x = ((25, 45, 15), (20, 45, 5), (10, 45, 10), (35, 45, 50)) $\in$ X

and $N = \begin{bmatrix} (10,45,10) & (45,45,45) & (40,45,30) & (15,45,25) \\ (50,45,30) & (5,45,20) & (20,45,5) & (40,45,30) \\ (45,45,45) & (15,45,25) & (40,45,30) & (35,45,50) \\ (5,45,20) & (15,45,25) & (40,45,30) & (20,45,5) \end{bmatrix}$

$\in$ M. We first find c.n max product {(x, N)} = ((30, 45,50), (45, 45, 45), (25,45, 15), (45, 45, 45)) = y_1 (say).

We see c-n max product $\{y_1, N\}$ = ((50, 45, 35), (45, 45, 45), (45, 45, 45), (50, 45, 35)) = y_2 (say).

c.n max product $\{(y_2, N)\}$ = ((50, 45, 35), (50, 45, 35), (40, 45, 30), (40, 35, 30) = y_3 (say).

c.n max-product $\{y_3, N\}$ = ((40, 45, 30), (50, 45, 35), (20, 45, 5), (35, 45, 50) = y_4 (say).

Now we find c-n max product $\{y_4, N\}$ = ((25, 45, 15), (40, 45, 30), (30, 45, 40), (50, 45, 35)) = y_5 (say)

c-n max product $\{y_5, N\}$ = {(30, 45, 40) (50, 45, 35), (45, 45, 45), (45, 45, 45)} = y_6 (say)

c-n max product $\{y_6, N\}$ = ((45, 45, 45), (30, 45, 40), (45, 45, 45), (35, 45, 50)) = y_7 (say).

Now c.n max product $\{y_7, N\}$ = ((45, 45, 45), (45, 45, 45), (50, 45, 35), (45, 45, 45)) = y_8 (say).

c.n max product $\{y_8, N\}$ = ((50, 45, 35), (45, 45, 45), (40, 45, 30), (45, 45, 45)) = y_9 (say).

We find the c.n max prod. $\{y_9, N\}$ = ((50, 45, 35), (50, 45, 35), (40, 45, 30), (40, 45,30)) = y_{10} (say).

c.n max product $\{y_{10}, N\}$ = ((40, 45, 30), (50, 45, 35), (20, 45, 5) (35, 45, 50)) = y_{11} (say).

We now find c.n max product $\{y_{11}, N\}$ = (25, 45, 15), (40, 45, 30), (30, 45, 40), (50, 45, 35)) = y_{12} (say).

We see $y_{12} = y_5$.

Thus this does not converge to a fixed point however the resultant is a limit cycle given by

$$x \to y_1 \to y_2 \to y_3 \to y_4 \to y_5 \to y_6 \to \ldots \to y_{12} = y_5.$$

Thus the product ends in a limit cycle just after 12 iterations.

Now consider

a = ((15, 45, 25), (25, 45, 15), (5, 45, 20), (20, 45, 5)) $\in$ X.

We find c.n max-product {a, N} = ((45, 45, 45), (25, 45, 15), (50, 45, 35), (15, 45, 25)) = p_1 (say).

c.n max-product {p_1, N} = ((50, 45, 35) (45, 45, 45) (50, 45, 35), (45, 45, 45)) = p_2 (say).

c.n max-product {p_2, N} = ((50, 45, 35), (50, 45, 35), (40, 45, 30), (45, 45, 45)) = p_3 (say).

c.n max-product {p_3, N} = ((40, 45, 30), (50, 45, 35), (40, 45, 30),(35, 45, 50)) = p_4 (say).

c.n max-product {p_4, N} = ((40, 45, 30), (50, 45, 35), (25, 45, 15), (40, 45, 30)) = p_5 (say).

c.n max-product {p_5, N} = ((40, 45, 30), (50, 45, 35), (10, 45,10),(50, 45, 35)} = p_6 (say).

c.n max-product {p_6, N} = ((30, 45, 40), (40, 45, 30), (20, 45, 5), (50, 45, 35)) = p_7 (say).

c.n max-product {p_7, N} = ((30, 45, 40), (35, 45, 50), (45, 45, 45), (40, 45, 30) = p_8 (say).

c.n max-product {p_8, N} = ((45, 45,45) (50, 45, 35), (45, 45, 45), (35, 45, 50)) – p_9 (say).

Now we find cn max-product {p_9, N} = ((45, 45, 45), (45, 45, 45), (40, 45, 30), (40, 45, 30)) = p_{10} (say).

The c.n max-product of $\{p_{10}, N\}$ = ((50, 45,35), (50, 45, 35), (40, 45, 30), (40, 45, 30)) = p_{11} say)

c.n max product $\{p_{11}, N\}$ = ((40, 45, 30), (50, 45, 35), (20, 45, 5), (35, 45, 50)} = p_{12} (say)

Certainly as both the sets X and M contains only a finite number of elements we will after a finite number of iterations arrive at a fixed point or a limit cycle.

Next we find this type of operations on $m \times n$ matrices of neutrosophic triplet groups $m \neq n$.

We show how in this case we will arrive at a fixed point pair or a limit cycle pair. This type of c.n max-product will be used in the NtgCMs model which is defined in chapter four.

Example 3.11. Let S = $\{Z_{35}, \times\}$ be the semigroup under product modulo 35.

The neutral elements (idempotents) of Z_{35} are 15 and 21.

The neutrosophic triplet groups of the neutral element 15 are

A = {(5,15, 10), (10, 15, 15), (25, 15, 30), (30, 15, 25), (20, 15, 20), (15,15,15)}. The neutrosophic triplet groups of the neutral element 21 are

B = {(7, 21, 28), (28, 21, 7), (14, 21, 14), (21, 21, 21)}.

Let M = $\left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \end{bmatrix} \middle/ a_i \in A; 1 \le i \le 10\right\}$

be the collection of all 5 × 2 neutrosophic triplet group matrices with entries from A.

Let X = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in A, 1 \le i \le 5\}$ be the set of all row vectors or row matrices of neutrosophic triplet groups.

Y = $\{(a_1, a_2) / A_i \in A; 1 \le i \le 2\}$ be the row vector or row matrices of neutrosophic triplet groups.

Now we define both c.n max min and c.n max product operations using elements of X, Y and M.

Let P = $\begin{bmatrix} (5,15,10) & (20,15,20) \\ (15,15,15) & (10,15,5) \\ (30,15,25) & (15,15,15) \\ (25,15,30) & (15,15,15) \\ (15,15,15) & (10,15,5) \end{bmatrix} \in$ M.

Let x = ((15, 15, 15), (5, 15, 10), (30, 15, 25), (10, 15, 5), (15, 15, 15)) ∈ ×.

We find c.n max products {x, P} = ((25, 15, 30), (30, 15, 25)) = y_1 (say).

Now we define P^t =

$$\begin{bmatrix} (5,15,10) & (15,15,15) & (30,15,25) & (25,15,30) & (15,15,15) \\ (20,15,20) & (10,15,5) & (15,15,15) & (15,15,15) & (10,15,15) \end{bmatrix}$$

We calculate c.n max product (y_1, P^t) = ((20, 15, 20), (25, 15, 30), (30, 15, 25), (30, 15, 25), (25, 15, 30)) = x_1 (say).

Now we find c.n max prod {x_1, P}

= ((30, 15, 5), (30, 15, 25)) = y_2 (say)

We get c.n max prod {y_2 ,P^t} = ((10,15, 5), (30, 15, 25) (30, 15, 25), (30, 15, 25), (30, 15, 25)) = x_2 (say)

c.n max prod {x_2 ,P}

= ((30, 15, 25) (30, 15, 25)) = y_3 (say).

We see c.n max product {y_3 ,P^t} = x_3 (= x_2)

Thus this converges to a fixed point pair given by {((10, 15, 5), (30, 15, 25), (30, 15, 25), (30, 15, 25), (30, 15, 25)), ((30, 15, 25), (30, 15, 25)}}. I

Now using the same x and the same P we now calculate

c.n max min {x, P};

c.n max min {x, P} = ((30, 15, 25), (15, 15, 15)) = y_1 (say).

c.n max min {y_1,P^t}= ((15, 15,15) (15, 15, 15), (30, 15, 25), (25, 15, 30), (15, 15, 15)) = x_1 (say).

c.n max min {x_1, P} = ((30, 15, 25), (15,15, 15)) = y_2 (say) = y_1

c.n max min {y_2 P^t} = c.n max min {y_1, P^t} = x_1.

Thus in this case also we see the row vector converges to a fixed point pair given by {(15, 15, 15), (15, 15, 15), (30, 15, 25), (25, 15, 30), (15, 15,15)), ((30,15, 25), (15, 15, 15))} II

We see I and II are different. That is the fixed point pair given by c.n max product is different from the fixed point pair given by c.n max min for the same row vector x of X on the same matrix P of M.

It is left as an open problem for the reader to find whether there exist a row vector in X such that for a given fixed P in M the row vector converges to the same fixed pair point both under the c.n max min as well as under the c.n max prod.

We next show some more interesting results on these neutrosophic triplet groups matrices.

Example 3.12 Let S = $\{Z_{39}, \times\}$ be the semigroup under product modulo 39. The neutral elements (idempotents) associated with Z_{39} are 13 and 27.

The neutrosophic triplets associated with the neutral element 27 are

A = {(3, 27, 9), (9, 27, 3), (6, 27, 24), (24, 27, 6), (18, 27, 21), (21, 27, 18), (15 27, 33), (33, 27, 15), (27, 27, 27), (12, 27, 12), (30, 27, 36), (36, 27, 30)}.

Clearly A is a group under product modulo 39.

Now let M = $\left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{pmatrix} / a_i \in A, 1 \leq i \leq 12\right\}$

be the collection of all matrices of neutrosophic triplet group.

Clearly $o(M) = 12^{12}$.

Let $X = \{(a_1, a_2, a_3) / a_i \in A, 1 \leq i \leq 3\}$ be the collection of all neutrosophic triplet groups row matrices with entries from A. X is a finite commutative group of order 12^3.

Let $Y = \{(a_1, a_2, a_3, a_4) / a_i \in A; 1 \leq i \leq 4\}$ be the row vector or row matrices of neutrosophic triplet groups entries from A.

Clearly $o(Y) = 12^4$ and Y is a group.

We now find using the matrix of neutrosophic triplet groups B from M and a $x \in X$ the c.n max min $\{x, B\}$ and

c.n max product $\{x, B\}$ where

$$B = \begin{bmatrix} (30,27,36) & (3,27,9) & (9,27,3) & (27,27,27) \\ (9,27,3) & (27,27,27) & (12,27,12) & (6,27,24) \\ (24,27,6) & (15,27,33) & (30,27,36) & (3,27,9) \end{bmatrix} \in M.$$

We first find c.n max min (x, B), c.n max min (x, B) = ((27, 27, 27), (6, 27, 24), (9, 27, 3), (27, 27, 27)) = y_1 (say).

$$\text{We see } B^t = \begin{bmatrix} (30,27,36) & (9,27,3) & (24,27,6) \\ (3,27,9) & (27,27,27) & (15,27,33) \\ (9,27,3) & (12,27,12) & (30,27,36) \\ (27,27,27) & (6,27,24) & (3,27,9) \end{bmatrix}.$$

We now find c.n max min $\{y, B^t\}$ = ((27, 27, 27), (9, 27, 3), (24, 27, 6)) = x_1 (say).

We now calculate c.n max min $\{x_1, B\}$ = ((27, 27, 27), (15, 27, 33), (24, 27, 6), (27, 27, 27)) = y_2 (say).

c.n max min $\{y_2\, B^t)$ = ((27, 27, 27), (15, 27, 33), (24,27, 6)) = x_2 (say).

c.n max min $\{x_2\, B\}$ = ((27, 27, 27), (15, 27, 33), (24, 26, 6), (27, 27, 27)) = y_3 (say).

Clearly $y_2 = y_3$ so this yields a fixed point pair given by {((27, 27, 27), (15, 27, 33), (24, 27, 6)), ((27, 27, 27), (15,27, 33), (24, 27, 6), (27, 27, 27))} I

Now for the same x and B we calculate

c.n max-product {x, B}.

c.n max-product {x, B} = ((30, 27, 36), (12, 27, 12), (36, 27, 30), (27, 27, 27)) = y_1 (say).

c.n max-product $\{y_1, B^t\}$ = ((36, 27, 30), (36, 27, 30), (27, 27, 27)) = x_1 (say).

c.n max-product {x, B} = ((27, 27, 27), (36, 27, 30), (30, 27, 36), (36, 27, 30)) = y_2 (say).

We now find max product $\{y_2, B\}$ = ((36, 27, 30) (36, 27, 30), (33, 27, 15)) = x_2 (say).

c.n max-product $\{x_2\, ,B\}$ = (27, 27, 27), (36, 26, 30), (15, 27, 33), (36, 27, 30)) = y_3 (say).

c.n max-product $(y_3 B^t\}$ = ((36, 27, 30), (36, 27, 30), (33, 27, 15)) x_3 (say) .

c.n max-product $\{x_3, B\}$= ((30, 27, 36), (36, 27, 30), (15, 27, 33), (36, 27, 30)) = y_4 (say).

c.n max-product (y_4, B^t) = ((36, 27, 30), (36, 27, 30), 933, 27, 15)) = x_4 (say).

c.n max product $\{x_4, B\}$= ((27, 27, 27), (36, 27, 30), (27, 27, 27), (36, 27, 30)) = y_5 (say).

c.n max product $\{y_5, B^t\}$ = ((36, 27, 30), (36, 27, 30), (33, 27, 15)) = x_5 (say).

We see $x_5 = x_4$ so the resultant is a fixed point pair given by {((27, 27, 27), (36, 27, 30), (27, 27, 27), (36, 27, 30)), (36, 27, 30), (36, 27, 30), (33, 27, 15)) II

Clearly I and II are distinct so the c.n max-min and c.n max - product yields different values however in this case both the methods yield only fixed point pair.

We now show how one can have a different type of product using a n × n square matrix and row vector with entries from the group of neutrosophic triplet groups.

If D is a n × n matrix and x is a 1 × n row vector we find $xD = y_1$ (say) then find $y_1D^t = x_1$ say then $x_1D = y_2$(say) next find y_2D^t so on until we arrive at a fixed point pair or a limit cycle pair.

We will illustrate this by an example and show how it is different from the usual fixed point worked out using the procedure $xD = y_1$ (say), $y_1D = y_2$ (say) and so on $y_tD = y_{t+1} = y_t$.

This procedure will only yield a fixed point or a limit cycle not a fixed point pair or a limit cycle pair.

Example 3.13. Let $S_1 = \{Z_{51}, \times\}$ be the semigroup under product modulo 51.

The neutral elements (or idempotents) of Z_{51} are 18 and 34.

We first give the neutrosophic triplet groups associated with 18.

A = {(3,18, 6), (6, 18, 3), (9, 18, 36), (27,18, 12), (30,18,21), (36, 18, 9), (12, 18, 27), (21, 18, 30), (39, 18, 24), (15, 18, 42),(45, 18, 48), (24, 18, 39), (42, 18, 15), (48, 18, 45), (33, 18, 33) (18,18,18)}.

Let B denote the collection of all neutrosophic triplet groups associated with 34.

B = {(34, 34, 34), (17, 34, 17)}.

Thus A is a cyclic group of order 16 and B is a cyclic group of order two.

We now consider M = {collection of all 4 × 4 matrices

with entries from A} = $\left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{pmatrix} \right.$ / $a_i \in A$,

$1 \leq i \leq 16\}$ be the collection of all 4 × 4 matrices of neutrosophic triplet groups.

Let X = {(a_1, a_2, a_3, a_4) / $a_i \in$ A; $1 \leq i \leq 4$} be the collection of all row matrices of neutrosophic triplet groups.

Let S =

$$\begin{bmatrix} (3,18,6) & (18,18,18) & (33,18,33) & (3,18,6) \\ (6,18,3) & (9,18,36) & (6,18,3) & (18,18,18) \\ (15,18,42) & (3,18,6) & (18,18,18) & (33,18,33) \\ (18,18,18) & (33,18,33) & (9,18,36) & (9,18,36) \end{bmatrix} \in M$$

and let x = ((3, 18,6), (9, 18, 36), (33, 18, 33),(18, 18, 18)) $\in$ X.

We find c.n max product {x, S} and c.n max min {x, S} using the transpose of S also.

Now c.n max product of {x, S} = ((36, 18, 9), (48, 18, 45), (48, 18, 45), (18, 18, 18)) = y_1 (say).

$$S^t = \begin{bmatrix} (3,18,6) & (6,18,3) & (15,18,42) & (18,18,18) \\ (18,18,18) & (9,18,36) & (3,18,6) & (33,18,33) \\ (33,18,33) & (6,18,3) & (18,18,18) & (9,18,36) \\ (3,18,6) & (18,18,18) & (33,18,33) & (9,18,36) \end{bmatrix} \in M.$$

We find c.n max product {y_1, S^t} = ((48, 18, 45), (33, 18, 33), (48,18, 45), (36, 18, 9)) = y_2 (say).

We find c.n max product {y_2, S} = ((45,18, 48), (48, 18, 45), (48, 18, 45), (48, 18, 45)) = y_3 (say).

c.n max product {y_3, S^t} = ((48, 18, 45), (48, 18, 45), (48, 18, 45), (45, 18, 48)) = y_4 (say).

c.n max product {y_4 ,S} = ((45, 18, 48), (48, 18, 45), (48, 18, 45), (48, 18, 45)) = y_5 (say).

Thus we see the value of the row vector x converges using M under c.n max product to a pair.

{((45, 18, 48), (48, 18,45), (48, 18, 45), (48, 18, 45)), ((48, 18, 45), (48,18, 45), (48, 18, 45), (45, 18, 48))} I

Now using the same pair x and S. We find the pair of resultant vectors using c.n max min operation.

c.n. max min {x, S} =((18, 18, 18), (18, 18, 18), (18, 18, 18), (33, 18, 33)) = y_1 (say)

c.n max min {y_1, S^t} = ((18, 18, 18), (18, 18, 18), (38, 18, 33), (18, 18, 18)) = y_2 (say).

We now find c.n max min (y_2S) = ((18, 18, 18) (18, 18, 18), (18, 18, 18), (33, 18, 33)) = y_3 (say).

Clearly $y_3 = y_1$; hence we see the vector converges or the resultant is a fixed point pair given by {((18, 18, 18), (18, 18, 18), (33, 18, 33), (18, 18, 18)), ((18, 18, 18), (18, 18, 18), (18, 18, 18), (33, 18, 33))} II

Clearly I and II are distinct. Further the operations c.n max product and c.n max min in this case yields only fixed point pairs. This type of working will find its applications in neutrosophic triplet groups relational maps model or in the neutrosophic triplet group Bidirectional Associative memories model.

The former structure will be defined and described in the following chapter.

The main advantage of using these neutrosophic triplet groups as entries is that once the element from the set is fixed automatically the neutral element and the anti element are fixed their by eliminating the arbitrariness present in the choice of elements.

Problems

In this section we propose some problems for the reader.

1. Find the idempotents of $\{Z_{226}, \times\} = S_1$ the semigroup under modulo product 226.

   i) Find all neutrosophic triplet groups associated with the neutral elements of Z_{226}.

   ii) Prove the neutrosophic triplet groups associated with the neutral elements forms a cyclic group

   iii) Find the order of them.

   iv) Find the generators of these cyclic groups.

2. Let $S = \{Z_{339}, \times\}$ be the semigroup under product modulo 339.

   i) Study questions (i) to (iv) of problem (1) for this S.

   ii) Compare S_1 of problem (1) with S of this problem.

3. Let $S = \{Z_{35}, \times\}$ be the semigroup under product .

Let A = {collection of all neutrosophic triplet groups associated with the neutral element 15}.

M = {collection of all 5 × 5 matrices with entries from A}. X = $\{(a_1, a_2, \ldots, a_5) / a_i \in A; 1 \le i \le 5\}$ be the row matrix of neutrosophic triplet groups.

If B =

$$\begin{bmatrix} (5,15,10) & (10,15,5) & (30,15,25) & (15,15,15) & (10,15,15) \\ (15,15,15) & (25,15,30) & (15,15,15) & (20,15,20) & (30,15,25) \\ (10,15,5) & (30,15,25) & (20,15,20) & (25,15,30) & (5,15,10) \\ (25,15,30) & (20,15,20) & (25,15,30) & (25,15,30) & (10,15,5) \\ (20,15,20) & (15,15,15) & (5,15,10) & (30,15,25) & (5,15,10) \end{bmatrix}$$

∈ M.

Let x = ((30, 15, 25), (5,15,10), (10, 15, 5) (20, 15, 20), (25,15, 30)) ∈ X.

i) Find the fixed point or the limit cycle associated with c.n max min {x, B}.

ii) Find the fixed point or the limit cycle associated with c.n max product {x, B}.

iii) Compare the resultants in (i) and (ii).

iv) Find all vectors a ∈ X which gives fixed points under c.n max product using this B.

v) Find all vectors x ∈ X which produce fixed points and c.n max min using the given B.

vi) Does the exists vectors b ∈ X which are fixed points under both c.n - max product and c.n max min?

vii) Can there be a $x \in X$ which gives the same fixed point both under c.n max min as well S c.n. max product?

viii) Can there be $x \in X$ which gives same limit cycles under both the c.n max min and cn max product ?

ix) Enumerate any other special features enjoyed by cn max product and cn max min.

4. Suppose $S = \{Z_n, \times\}$ be the semigroup under product $n = 2p$ (p a prime). A the set of neutrosophic triplet groups associated with S.

   M ={collection of all 5×3 matrices with entries from A}; Y = {1×5 matrices with entries from A} and X = {1×3 matrices with entries from A}. If c.n max min and c.n max product operations are performed for a fixed point pairs {x, P} where $x \in Y$ and $P \in M$.

   i) Does there exist any relation between the resultant vectors given by c.n max product and c.n max min?

   ii) Give a value for $x \in Y$ and a $B \in M$ such that one of c.n max product gives a fixed point pair whereas that of c.n max min yield a limit cycle pair or vice versa.

5. Let $S = \{Z_{143}, \times\}$ be the semigroup under product modulo 143.

   Let M = {collection of all 5×5 matrices with entries from A} where A is the larger of the two groups of

neutrosophic triplet groups associated with neutral elements of Z_{1453}.

i) Prove o(A) = 12.

ii) Prove o(M) = 12^{25}

iii) Let X = {(a_1 a_2 a_3 a_4 a_5) $a_i \in$ A; $1 \leq i \leq 5$}, prove o(X) = 12^5

iv) Using c.n max min {x, P} for x $\in$ X and P $\in$ M and the transpose of P find the limit point pair or the fixed point pair.

v) Give an example by choosing x $\in$ X and N $\in$ M such that the resultant pair of c.n matrix product {x, N} is a fixed point pair whereas the resultant pair using c.n max min yields a limit cycle pair.

6. Let S ={ Z_{194}, $\times$} be the semigroup under product modulo 194 and B = { Z_{291}, $\times$} be the semigroup under product modulo 291.

i) Prove S has a neutral element which yields 96 distinct neutrosophic triplet groups.

ii) Prove B also has a neutral element which yields 96 distinct neutrosophic triplet groups.

iii) Hence or otherwise prove the largest cardinality of the collection of all neutrosophic triplet groups can be the same even if the cardinality of the respective Z_n's may vary.

iv) Can we prove the conclusion in (iii) is from the fact that largest prime which divides 194 and 291 are the same?

v) Hence can we prove the cardinality of the group of neutrosophic elements is only dependent on the largeness of the prime which divides m and n of Z_m and Z_n and not of the fact m > n or n > m.

7. Determine a method by which all neutral elements (idempotents) can be found in Z_n; $n = p_1 p_2 \ldots p_t$; $t > 3$ and p_i are distinct primes; $1 \leq i \leq t$.

8. Let S ={Z_{2485}, ×} and B = {Z_{3995}, ×} be two semigroups under modulo product.

   i) Find all the neutral elements of S and B.

   ii) Which of the semigroups S or B has more number of classical group of neutrosophic triplet groups?

   iii) Which of the semigroups S or B has larger number of neutral elements?

   iv) Find all special and distinct features enjoyed by S and B.

9. Can we say if S is as in problem 8 the collection of all neutrosophic triplet groups associated with any neutral element will be a group?

10. Study problem (9) in case of S = {Z_{2310}, ×} and B = {Z_{4130}, ×} the semigroups under modulo product.

11. Do these neutral elements of S and B yield to classical group structure on the collection of neutrosophic triplet groups for any fixed neutral element?

12. Let $\{Z_n, \times\}$ be the semigroup under product modulo n, n a composite number. Does the collection of all neutral elements of Z_n form a semigroup or a group?

13. Let S = $\{Z_n, \times\}$ be a semigroup under product modulo n (n a composite number);

    i) When can we say S has neutral elements which contribute to duplets? (give condition on n).

    ii) When can we say Z_n has neutral elements which contribute to quasi neutrosophic triplet groups?

    iii) Characterize those n for which Z_n has quasi neutrosophic triplets groups.

    iv) If $r \in Z_n$ is a neutral element associated with quasi neutrosophic triplet groups than can r have with it some associated neutrosophic triplet groups? Justify.

14. Study questions (i) to (iv) of problem (13) in case of S = $\{Z_{120}, \times\}$.

15. Analyse the relation between quasi neutrosophic triplet groups and neutrosophic triplet groups.

16. If a neutral element in $\{Z_n, x\}$ gives duplets can the same neutral element yield quasi neutrosophic triplet groups and neutrosophic triplet groups?

17. Characterize those S = {Z_n, x} which can yield only duplets (for which n this can occur).

18. Characterize those n of S = {Z_n, x} for which the neutral elements can yield only quasi neutrosophic triplet groups.

19. Describe those n of S = {Z_n, x} which can yield only quasi neutrosophic triplet groups and neutrosophic triplet group and not duplets.

20. Characterize those n of S = {Z_n, ×} whose classical group of neutrosophic triplet groups can yield only fixed points for any s × s collection of square matrices and 1 × s row matrices with entries from that group.

    i) Does such a n exists?

    ii) If the collection of neutrosophic triplet groups is only a semigroup can we say fixed points alone is possible?

21. Characterize those n of S = {Z_n, ×} for which Z_n has neutrals whose associated collection of neutrosophic triplet groups forms only a semigroup.

22. Let S = {Z_n, ×}, where n = $2^8\ 3^5\ 5^2\ 7^3$ be the semigroup under product.

    i) Find all neutral elements of Z_n.

    ii) How many of these neutral elements contribute to the classical group of neutrosophic triplet groups?

iii) How many of these neutral elements contribute to semigroup of neutrosophic triplet groups?

iv) Does there exist neutrals associated with n which can contribute to duplets?

v) Does there exist neutrals associated with this n which can contribute to quasi neutrosophic triplet groups?

23. Let S = $\{Z_n, \times\}$ where n = 72 be the semigroup under product modulo n.

i) Find the neutral elements of 72.

ii) Which of these neutral elements contribute to neutrosophic triplet groups.

iii) Which has more number of neutrals which contribute to neutrosophic triplet group Z_{72} or Z_{69}?

iv) Can we say Z_{74} has more number of neutrosophic triplet groups than Z_{72} and Z_{69}?

v) Prove Z_{74} has only two neutrals 37 and 38.

vi) Show related with the neutral 38 has a collection of 36 nontrivial neutrosophic triplet groups.

vii) Prove these 36 elements from a cyclic group of order 36.

viii) Prove Z_{69} and Z_{72} has only classical groups of order certainly less than 36.

24. Can you show the largest classical group of neutrosophic triplet groups exists in Z_{pq} if p > q then there is a cyclic group of order (p – 1)?

25. If in S ={Z_{pqr}, ×} be the (p, q, r are distinct primes) semigroup under product modulo pqr. If p > q > r then will S have a classical cyclic group of order p – 1?

26. Let S = {Z_{30}, ×} be the semigroup under product verify problem 25 for this Z_{30}.

27. Let S = {Z_{385}, ×} be the semigroup under product modulo 385.

    a) Verify problem 25 for this Z_{385}.

    b) How many neutrals of Z_{385} contributes to neutrosophic triplet groups which are classical cyclic groups?

28. Enumerate all special features associated with neutrosophic triplet groups.

29. What are the probable applications of the neutrosophic triplet groups?

30. Calculate all the neutrals of Z_{2332}.

31. Which of the neutrals in Z_{2332} contribute to cyclic group of neutrosophic triplet groups.

32. Obtain all special features of Z_{p^n} ; p a prime n > 2 in terms of neutrals.

33. Find all neutrals of Z_{256}.

34. Find all neutrals of Z_{251}.

35. Prove these group of neutrosophic triplet groups can be used in the construction of mathematical models.

36. Can these neutrosophic triplet groups for some Z_n form only semigroup?

37. When these neutrosophic triplet groups for some Z_n is only a semigroup will these contribute to duplets?

38. Does there exists $S = \{Z_n, \times\}$ which has only neutrals which generate quasi neutrosophic triplet groups and not neutrosophic triplet groups?

39. Can the collection of quasi neutrosophic triplet groups for some neutrals in Z_n, for some n form a classical group? Justify.

Chapter Four

APPLICATIONS OF NEUTROSOPHIC TRIPLET GROUPS TO MATHEMATICAL MODELS

In this chapter we for the first time introduce the new notion of Neutrosophic Triplet Groups Cognitive Maps (NTGCMs) models, Neutrosophic Triplet Groups Relational Maps (NTGRMs) models and models which use soft computing principles like single layer feed forward network, multilayer feed forward network, perceptron etc. Instead of using on or off state of the Fuzzy Cognitive Maps (FCMs) model we can use the off state say (0, 0, 0) however for the on state can take any of the values from the neutrosophic triplet groups which will be known as the nodes or concepts of the newly built model akin to the FCMs model. We first introduce the Neutrosophic Triplet Groups Cognitive Maps model.

We however make the formal definition of Neutrosophic Triplet Groups Cognitive Maps (NTGCMs) model or (NtgCMs) model.

Let S = $\{Z_{2p}, \times\}$ be the semigroup under product modulo 2p, p an odd prime.

Let p + 1 and p be the neutral elements of Z_{2p}; p + 1 is a neutral element which contributes to neutrosophic triplet groups which are nontrivial. Infact associated with p + 1 are some p – 1 number of neutrosophic triplet groups which forms a classical group of order p – 1 which is also cyclic with p + 1 as its identity.

Let $C_1, C_2, \ldots, C_n$ be the nodes or concepts associated with some problem let them take values from the set B = {0, 2, 4, …, (p – 1) 2}.

Let us assume if C_i node is 0 then it is in the off state if it takes any of the other values from B \ {0} the node is in the on state with appropriate property or that is the value of the node.

Here if the nodes C_i to C_j has an impact depending on the value of impact (or effect of one node on the other) a weight is given from the set B \ {0} if there is no impact of C_i into C_j then the weight is 0. If the weight is a non zero then depending on the impact a value from B \ {0} is given.

Here it is pertinent to make the following two observations:

i) The nodes / concepts can take 0 or any value from B \ {0}, however in case of FCMs they can take only values on or off in on value 1 and off value 0.

ii) In case of FCMs two C_i has impact on C_j then 1 is given if increase in C_i increases C_j or decrease

in C_i decrease s C_j and –1 increase in C_j decreases C_j or decrease in C_i increases C_j. The value 0 is given if no effect of C_i is found on C_j.

iii) At the outset this NtgCMs or (NTGCMs) model which will be based on special type of MODFCMs model is better than FCMs model as we cannot always make an assumption that a state is always on state or off state and a on state has a value 1 for we can have a partial or a semi on state as these are cognitive models.

Any unit can function partially and not fully also the term partially is not a quantity for it can always have shades of values.

For instance take an industry if all units are fully functioning we can say 1 is the value it may so happen all units are closed then 0, however the security unit is always in on state so an industry on all days functions partially. Sometimes only security unit and dispatch unit may function. In some day (even on holidays) security unit and coordination committee unit or executive committee units may function and so on. If we have to say some t units and we have some n nodes /concepts we can give the on state or partial state or off state of these nodes with values from B.

In the opinion of the authors this way of defining is more appropriate than the usual way by which FCMs are defined.

Now we discuss about the weights. We see in case of FCMs the weight is 0 or 1 or – 1 but this is not always possible their may be a partial impact of a C_i onto a mode C_j how are

going to describe this situation. To this effect we give for weights the values from the set B.

We see certainly the MODCMs would be more practical than FCMs. We are also certain to arrive at a fixed point or a limit cycle after a finite number of iterations. This can be clearly proved as B is a finite set under modulo product 2q.

We can use 3 types of operations usual composition or max product or max min under all the three operations they will yield different answer.

We will illustrate all these facts by some examples.

Example 4.1. Let C_1, C_2, C_3, C_4, C_5 and C_6 be six nodes / concepts which take the node values from B = {0, 2, 4, 6, 8, 10, 12} $\subseteq Z_{14}$ be the collection. Clearly B \ {0} is a cyclic group of order six with 8 as its multiplicative identity.

The elements x = (a_1, a_2, ... ,a_6) takes its values from B, that is $a_i \in B$, $1 \leq i \leq 6$.

Thus if X = {(a_1, a_2, a_3, a_4, a_5, a_6) / $a_i \in B$, $1 \leq i \leq 6$} are the state vectors of MODCMs akin to the state vector in FCMs which can take values only 0 or 1 in the case of simple FCMs. So we can over come the crisp on or off state to partially on state, some what on state, just on state and so on. The weights also need not be 0 or 1, it can be any value from B.

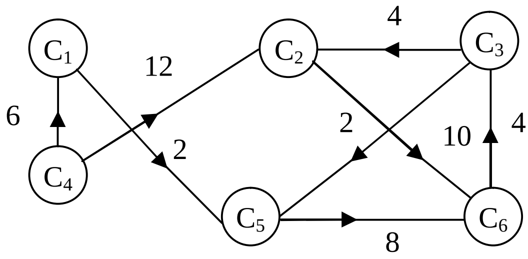

Figure 4.1

The directed MOD graph relating the six nodes whose weights are given above is described in the above figure.

Let M be the connection matrix associated with this above graph.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 4 & 0 & 0 & 2 & 0 \\ 6 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

M can also be called as the MOD connection matrix of the directed MOD graph.

Let as consider the initial state vector

$x = (0\ 2\ 0\ 6\ 0\ 0) \in X$ that is C_1, C_3, C_5 and C_6 states are off and C_2 takes the state 2 and C_4 has taken its state value as 6.

Now we find the effect of x on M.

xM = (8, 2, 0, 0, 0, 6) we do not update the vector xM by replacing the original state values of C_2 and C_4 as in MODCMs can come to off state or any other partial state in the due course of time. Only xM = (8, 2, 0, 0, 0, 6) = y(say).

Now we find the effect of y_1 on M.

$y_1M = (0, 0, 10, 0, 2, 6) = y_2$ (say)

$y_2M = (0, 12, 10, 0, 6, 2) = y_3$ (say)

$y_3M = (0, 12, 8, 0, 6, 0) = y_4$ (say)

$y_4M = (0, 4, 0, 0, 2, 0) = y_5$ (say)

$y_5M = (0, 0, 0, 0, 0, 6) = y_6$ (say)

$y_6M = (0, 0, 10, 0, 0, 0) = y_7$ (say)

$y_7M = (0, 12, 0, 0, 6, 0) = y_8$ (say)

$y_8M = (0, 0, 0, 0, 0, 0) = y_9$ (say).

So the partial on state of the nodes C_2 and C_4 alone can lead to the off state of all nodes also if no updating is done.

Let x = (2, 0, 2, 0, 0, 4) $\in$ X that is nodes C_1, C_3 and C_6 take some values and the nodes C_2, C_4 and C_5 take only off state.

The effect of x on M is given by

$xM = (0, 8, 2, 0, 8, 0) = y_1$ (say)

$y_1M = (0, 8, 0, 0, 4, 4) = y_2$ (say)

$y_2M = (0, 0, 2, 0, 0, 0) = y_3$ say

$y_3M = (0, 8, 0, 0, 4, 0) = y_4$ say

$y_4M = (0, 0, 0, 0, 0, 0) = y_5$ gives only zero state.

So a natural question would be will all on state of all nodes lead to zero states. If such a thing occurs the matrix and the graph can be categorized as special type of graphs and hence they can find their applications in special set up.

Now let $x = (2, 2, 0,4, 2, 6) \in X$.

We find the effect of x on M

$xM = (10, 6, 10, 0, 4, 8) = y_1$ (say)

$y_1M = (0, 12, 4, 0, 12, 8) = y_2$ (say)

$y_2M = (0, 2, 4, 0, 8, 6) = y_3$ (say)

$y_3M = (0, 2, 10, 0, 8, 0) = y_4$ (say)

$y_4M = (0, 12, 0, 0, 6, 0) = y_5$ (say)

$y_5M = (0, 0, 0, 0, 0, 0)$.

So the reader is left with the task of studying such structures.

Now for the same M we use max product operation and find the resultant.

Max product $(x, M) = (8, 2, 0, 0, 0, 0) = y_1$

Max product $(y_1, M) = (0, 0, 0, 0, 4, 6) = y_2$ (say)

Max product $(y_2, M) = (0, 0, 10, 0, 0, 4) = y_3$ (say)

Max product $(y_3, M) = (0, 12, 2, 0, 6, 0) = y_4$ (say)

Max product $(y_4, M) = (0, 8,0, 0, 4, 0) = y_5$ (say)

Max product $(y_5, M) = (0, 0, 0, 0, 0, 0)$.

So now we are forced to take some modification in the first place the automatic method of updating of the non zero state of the nodes can make a chaos so we take up the method in which the state vectors at each stage should be updated.

Secondly we propose a conjecture or open problem.

Conjecture 4.1. Let $S = \{Z_{2p}, \times\}$ be the semigroup under product.

Let $B = \{0, 2, 4, \ldots, 2(p – 1)\}$ be the cyclic group with $p + 1$ as the identity with respect to product.

Suppose M is a $n \times n$ square matrix with entries from B and if $X = \{(a_1, \ldots, a_n) / a_i \in B\}$ $1 \leq i \leq n$ can we say every $x \in X$ is such that $xM = y_1 \in X$, $y_1M = y_2 \in X$ and so on after a t^{th} stage $y_tM = (0, 0, \ldots, 0)$? Further such types are more mathematically interesting and happens to be a challenging problem.

If not under what conditions on M such things happen.

If $y_tM = (0, 0, \ldots, 0)$ after that t^{th} iteration can we say even under max product we arrive at $(0, 0, \ldots, 0)$.

Will this be true in case of max min $\{x, M\}$ also.

Characterize those special $n \times n$ matrices M.

Now we proceed onto discuss the same problem with updating of the vectors C_2 and C_4 at each stage and analyze whether the final resultant on M be x itself or a different state vectors.

Consider

$$xM = (8, 2, 0, 0, 0, 6) \xrightarrow[\text{updating}]{} (8, 2, 0, 6, 0, 6) = y_1 \text{ say}$$

$y_1M = (8, 2, 10, 0, 2, 6) \to (8, 2, 10, 6, 2, 6) = y_2$ (say)

$y_2M = (8, 0, 10, 0, 8, 8) \to (8, 2, 10, 6, 8, 8) = y_3$ (say)

$y_3M = (8, 0, 4, 0, 8, 0) \to (8, 2, 4, 6, 8, 0) = y_4$ (say)

$y_4M = (8, 4, 0, 0, 10, 0) \to (8, 4, 0, 6, 10, 0) = y_5$ (say)

$y_5M = (8, 2, 0,0, 2, 8) \to (8, 2, 0, 6, 2, 8) = y_6$ (say)

$y_6M = (8, 2, 4, 0, 2, 8) \to (8, 2, 4, 6, 2, 8) = y_7$ (say)

$y_7M = (8, 4, 6, 0, 10, 8) \to (8, 4, 6, 6, 10, 8) = y_8$ (say)

$y_8M = (8, 0, 4, 0, 0, 8) \to (8, 2, 4, 6, 0, 8) = y_9$ (say)

$y_9M = (8, 4, 4, 0, 10, 6) \to (8, 4, 4, 6, 10, 6) = y_{10}$ (say)

$y_{10}M = (8, 0, 10, 0, 10, 0) \to (8, 2, 10, 6, 10, 0) = y_{11}$ (say)

$y_{11}M = (8, 0, 0, 0, 8, 8) \to (8, 2, 0, 6, 8, 8) = y_{12}$ (say)

$y_{12}M = (8, 2, 4, 0, 2, 0) \to (8, 2, 4, 6, 2, 0) = y_{13}$ (say)

$y_{13}M = (8, 2, 0, 0, 10, 8) \to (8, 2, 0, 6, 10, 8) = y_{14}$ (say)

$y_{14}M = (8, 2, 4, 0, 2, 2) \to (8, 2, 4, 6, 2, 2) = y_{15}$ (say)

$y_{15}M = (8, 4, 8, 0, 10, 8) \to (8, 4, 8, 6, 10, 8) = y_{16}$ (say)

$y_{16}M = (8, 6, 4, 0, 4, 10) \to (8, 6, 4, 6, 4, 10) = y_{17}$ (say)

$y_{17}M = (8, 4, 12, 0, 10, 8) \to (8, 4, 12, 6, 10, 8) = y_{18}$ (say)

$y_{18}M = (8, 8, 4, 0, 12, 8) \to (8, 8, 4, 6, 12, 8) = y_{19}$ (say)

$y_{19}M = (8, 4, 4, 0, 10, 8) \to (8, 4, 4, 6, 10, 8) = y_{20}$ (say)

$y_{20}M = (8, 4, 4, 0,10, 8) \to (8, 4, 4, 6, 10, 8) = y_{21}$ (say).

Clearly $y_{21} = y_{20}$ so the limit point is a fixed point.

Hence it has become mandatory to update the nonzero state at each and every stage to see to that the resultant does not crumble to a zero state.

Next we find for the same x and M max product {x, M}.

max product $\{x, M\} = (8, 2, 0, 0, 0, 0) \to (8, 2, 0, 6, 0, 0)$

$= y_1$ (say)

max product $\{y_1, M\} = (8, 2,0, 0, 2, 6) \to (8, 2, 0, 6, 2, 6)$

$= y_2$ (say)

max product $\{y_2, M\} = (8, 2, 10, 0, 2, 8) \to$

$(8, 2, 10, 6, 2, 8) = y_3$ (say)

max product $\{y_3, M\} = (8, 0, 4, 0, 8, 8) \to$

$(8, 2, 4, 6, 8, 8) = y_4$ (say)

max product $\{y_4, M\} = (8, 4, 4, 0, 10, 0) \to$

$(8, 4, 4, 6, 10, 0) = y_5$ (say)

y_5 max product $\{y_5, M\} = (8, 4, 0, 0, 10, 8) \to$

$(8, 4, 0, 6, 10, 8) = y_6$ (say)

max product $\{y_6, M\} = (8, 2, 4, 0, 2, 8) \to$

$(8, 2, 4, 6, 2, 8) = y_7$

max product $(y_7, M) = (8, 4, 4, 0, 10, 8) \to$

$(8, 4, 4, 6, 10, 8) = y_8$.

max product $\{y_8, M\} = (8, 4, 4, 0, 10, 8) \to$

$(8, 4, 4, 6, 10, 8) = y_9$ (say) II

It is clear $y_9 = y_8$ so the resultant vector is a MOD fixed point.

It is surprising to see under both operations usual multiplication and under max product in this case the resultant happens to be the same.

We leave it as a problem to the reader to find whether there are vectors $x \in X$ such that xM gives a resultant state vector different from that of max product.

Now we work with the maximum {x, M} for the same x and M

max min $\{x, M\} = (6, 6, 0, 0, 0, 2) \to (6, 6, 0, 6, 0, 2) = y_1$

max min $\{y_1, M\} = (6, 6, 2, 0, 2, 6) \to (6, 6, 2, 6, 2, 6) = y_2$

max min $\{y_2, M\} = \{6, 6, 4, 0, 2, 6) \to (6, 6, 4, 6, 2, 6) = y_3$

max min $\{y_3, M\} = (6, 6, 4, 0, 2, 6) \to (6, 6, 4, 6, 2, 6)$

$= y_4$ (say)

Clearly $y_4 = y_3$ thus the resultant vector is a fixed point given (6, 6, 4, 6, 2, 6) which is different from other resultant state vectors.

Thus we see with on state of nodes C_2 and C_4 leads to the non zero state of all other nodes in the resultant to get a non zero state.

The example is not any real world problem or from any real world data.

Next we proceed onto describe the notion of neutrosophic triplet group cognitive maps model in the following.

Let the graph be the same as for the MOD cognitive models.

Now each edge weight is transformed into a neutrosophic triplet group for a = 12, neut 12 = 8 and anti 12 = 10 so in the place of weight 12 we replace it by the neutrosophic triplet group weight (12, 8, 10).

Likewise the weight of the edge $\xrightarrow[C_5C_6]{}$ which is 8 is changed to (8, 8, 8) and that of the weight of the edge $\xrightarrow[C_6C_3]{}$ which is 4 is changed to (4, 8, 2) and so on.

Thus the MOD directed graph is transformed to neutrosophic triplet group graph which is described by the following Figure 4.2.

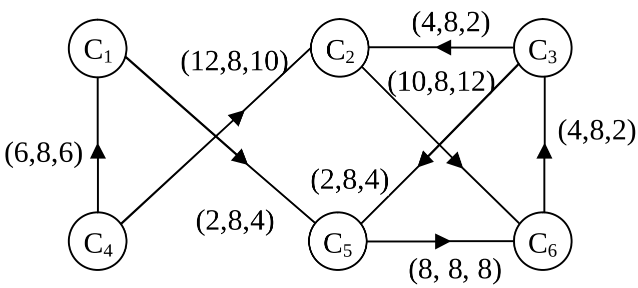

Figure 4.2

Let N be the connection neutrosophic triplet group matrix N of the directed neutrosophic triplet group graph given in Figure 4.2.

$$N = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 \quad\quad & C_2 \quad\quad & C_3 \end{array} \\ \begin{bmatrix} (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (4,8,2) & (0,0,0) \\ (6,8,6) & (12,8,10) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (4,8,2) \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} C_4 & C_5 & C_6 \end{array}$$
$$\left.\begin{array}{ccc}
(0,0,0) & (2,8,4) & (0,0,0) \\
(0,0,0) & (0,0,0) & (10,8,12) \\
(0,0,0) & (2,8,4) & (0,0,0) \\
(0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (8,8,8) \\
(0,0,0) & (0,0,0) & (0,0,0)
\end{array}\right]$$

In view of all these we first define the notion of neutrosophic triplet groups directed graph in the following.

Definition 4.1. *A simple graph G with {v_1, …, v_n} as its vertex set is defined to be a neutrosophic triplet group graph if the edge weights are from the neutrosophic triplet group collection associated with one neutral element from Z_m, m a composite number.*

We will first illustrate this situation by some examples.

Example 4.2. Let G be a graph with vertex set v_1, v_2, v_3, v_4 and v_5. The edge weights are from the set.

B = {(0, 0, 0), (18, 18, 18), (2, 18, 26), (26, 18, 2), (4, 18, 30), (30, 18, 4), (8, 18, 32), (32, 18, 8), (16, 18, 16), (6, 18, 20), (20, 18, 6), (12, 18, 10), (10, 18, 12), (24, 18, 22), (22, 18, 24), (14, 18, 28), (28, 18, 14)} related to the neutral 18 of Z_{34}.

The neutrosophic triplet group graph with edge weights from B with v_1, v_2, v_3, v_4 and v_5 is as follows.

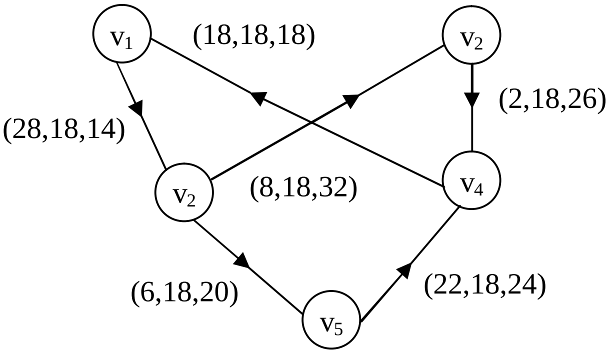

Figure 4.3

The matrix V related to the graph with neutrosophic triplet group edge weight is as follows:

$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \quad v_1 \quad & \quad v_2 \quad & \quad v_3 \quad & \quad v_4 \quad & \quad v_5 \quad \end{array} \\
\begin{bmatrix}
(0,0,0) & (0,0,0) & (28,18,14) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (0,0,0) & (2,18,26) & (0,0,0) \\
(0,0,0) & (18,18,32) & (0,0,0) & (0,0,0) & (6,18,20) \\
(18,18,18) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (0,0,0) & (22,18,24) & (0,0,0)
\end{bmatrix}
\end{array}
$$

We see the graph given in Figure 4.3 is a simple directed graph.

We now given an example of a simple graph with loops.

Example 4.3. Let G be a simple graph with loops associated with the vertex set $v_1, v_2, \ldots, v_7$ and edge weights taken from the set B given in example 4.2. G is defined as the neutrosophic triplet group (simple graph with loops which is given by the following Figure 4.4.

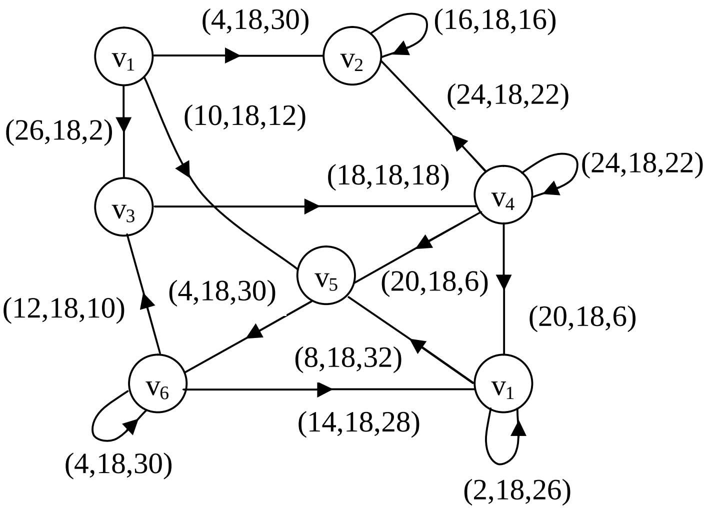

Figure 4.4

Now let W be the matrix of neutrosophic triplet group associated with the simple neutrosophic triplet group graph with loops.

Now we find the neutrosophic triplet group matrix M associated with this neutrosophic triplet group graph

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc}
(0,0,0) & (4,18,30) & (26,18,2) & (0,0,0) & (10,18,12) & (0,0,0) & (0,0,0) \\
(0,0,0) & (16,18,16) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (0,0,0) & (18,18,18) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (24,18,22) & (0,0,0) & (28,18,14) & (20,18,6) & (0,0,0) & (20,18,6) \\
(0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (12,18,10) & (0,0,0) & (4,18,30) & (4,18,30) & (14,18,28) \\
(0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (8,18,32) & (0,0,0) & (2,18,26)
\end{array}\right] \end{array}$$

Thus all the neutrosophic triplet groups graphs given so far are only directed one so they are not symmetric about the diagonal. We now give an example of the simple neutrosophic triplet graphs which are not directed by an example.

Example 4.4. Let v_1, v_2, v_3, v_4, v_5, v_6,v_7 and v_8 be the set of vertices of the neutrosophic triplet group graphs with edge weights from B of example given by the following Figure 4.5.

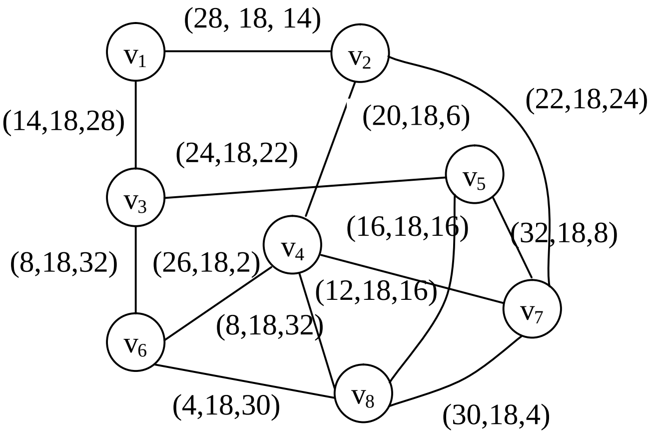

Figure 4.5

Now we give the related neutrosophic triplet groups matrix associated with the simple neutrosophic triplet group graphs.

$$
M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \quad v_1 \quad & \quad v_2 \quad & \quad v_3 \quad & \quad v_4 \quad \end{array} \\
\left[\begin{array}{cccc}
(0,0,0) & (28,18,14) & (14,18,24) & (0,0,0) \\
(28,18,14) & (0,0,0) & (0,0,0) & (24,18,6) \\
(14,18,28) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (20,18,6) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (24,18,22) & (0,0,0) \\
(0,0,0) & (0,0,0) & (8,18,32) & (26,18,2) \\
(0,0,0) & (22,18,24) & (0,0,0) & (16,18,16) \\
(0,0,0) & (0,0,0) & (0,0,0) & (8,18,32)
\end{array} \right. \end{array}
$$

$$\begin{array}{cccc}
v_5 & v_6 & v_7 & v_8 \\
\end{array}$$
$$\left.\begin{array}{cccc}
(0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\
(0,0,0) & (0,0,0) & (22,18,24) & (0,0,0) \\
(24,18,22) & (8,18,32) & (0,0,0) & (0,0,0) \\
(0,0,0) & (26,18,2) & (16,18,16) & (8,18,32) \\
(0,0,0) & (0,0,0) & (32,18,8) & (16,18,16) \\
(0,0,0) & (0,0,0) & (0,0,0) & (4,18,36) \\
(32,18,8) & (0,0,0) & (0,0,0) & (30,18,14) \\
(16,18,16) & (4,18,30) & (30,18,14) & (0,0,0)
\end{array}\right]$$

We see the matrix M is symmetric about the main diagonal so we would be using the neutrosophic triplet groups graphs get the related neutrosophic triplet group matrix however using the matrix. We cannot define product of matrices only we can define the notion of max product or max min operations which is a necessity for us to define the notion of neutrosophic triplet groups Cognitive maps (ntg CMs) model.

We just describe these model in a line or two.

Throughout our discussion we may take a directed neutrosophic triplet groups graph with entries from a set of neutrosophic triplet groups associated with some neutral elements p + 1 of Z_{2p}, p an odd prime.

Clearly the collection of neutrosophic triplet groups forms a cyclic group of order p – 1 with p + 1 as the neutral element and p + 1 serves as the multiplicative identity. For the directed neutrosophic triplet groups graphs edge weights are from the collection of all neutrosophic triplet groups associated with neutral element p + 1 of Z_{2p}, p an odd prime.

Now $C_1, C_2, \ldots, C_n$ are neutrosophic triplet groups nodes that is a node C_i takes either the neutrosophic triplet group or is 0. We do not have the usual notion of on or off state. A state is either (a, b, c) = (a, neut(a), anti(a)) or zero yet another advantage is that even if $a \in 2Z_{2p} \setminus \{0\}$ is given we can map the rest of them viz. neut(a) and antia. Thus any MOD cognitive models with entries from $2Z_{2p} \setminus \{0\}$ is such that if $a \in 2Z_{2p} \setminus \{0\}$ is known automatically neut (a) and anti a are fixed.

Now on similar lines if the edge weights of the directed graphs are from $2Z_{2p} \setminus \{0\}$ then also the neutrosophic triplet groups graph can be determined.

We will assume in a neutrosophic triplet group cognitive maps model the nodes $C_1, C_2, \ldots, C_n$ can take values from the neutrosophic triplet groups associated with the neutral element p + 1 of Z_{2p} which are p – 1 in number and we adjoin (0, 0, 0) with it.

So the state vectors $X = \{(a_1, a_2, \ldots, a_n) / a_i \in$ {collection of neutrosophic triplet groups associated with p + 1}; $1 \leq i \leq n\}\}$.

M is the $n \times n$ neutrosophic triplet groups matrix with entries from the collection of all neutrosophic triplet groups got as a connection matrix of the graph of neutrosophic triplet groups with $C_1, C_2, \ldots, C_n$ as nodes or vertices or concepts.

It is pertinent to keep on record that in case of neutrosophic triplet groups matrices we cannot define the usual product only c.n mat-product and c.n max min operation can be performed without difficulty.

We will illustrate this situation by some examples.

Example 4.5. Let C_1, C_2, C_3, C_4 and C_5 be 5 district nodes or concepts. Let G be the directed neutrosophic triplet graph with edge weight from the set

B = {(0, 0, 0), (2, 14, 20), (20, 14, 2), (4, 4, 10), (10, 14, 4), (8, 14, 18), (18, 14, 8),(16, 14, 22), (22, 14, 16), (6, 14, 24), (24, 14, 6), (12, 14, 12), (14, 14, 14)} associated with the neutral 14 of Z_{26}.

G =

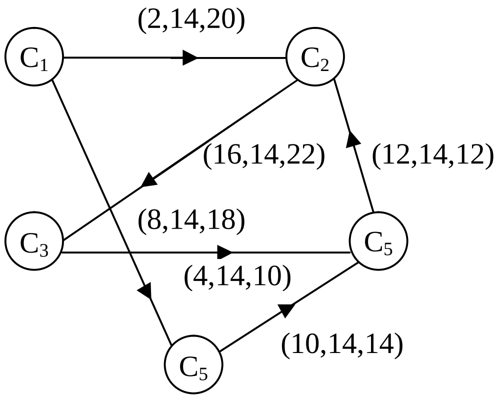

Figure 4.6

Now G is the directed neutrosophic triplet groups graph associated with the nodes C_1, C_2, …,C_5.

The neutrosophic triplet groups connection matrix M of the graph G is as follows.

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} (0,0,0) & (2,14,20) & (0,0,0) & (0,0,0) & (8,14,18) \\ (0,0,0) & (0,0,0) & (16,14,22) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (8,14,18) & (0,0,0) \\ (0,0,0) & (12,14,12) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (10,14,4) & (0,0,0) \end{bmatrix} \end{array}.$$

Let the collection of all state vectors associated with the dynamical system of neutrosophic triplet groups cognitive maps (NtgCMs) model be denoted by

$$X = \{(x_1, x_2, x_3, x_4, x_5) / x_i \in B;\ 1 \leq i \leq 5\}.$$

Let x = (((10, 14, 4), (0, 0, 0), (14, 14, 14), (2, 14, 20), (0, 0, 0)) $\in \times$.

We will find the effect of x on M c.n max product {x, M} = {((0, 0, 0), (24, 14, 6), (0, 0, 0), (8, 14, 18), (24, 14, 20))} $\xrightarrow{\text{after updating}}$ we get ((10, 14, 4), (24, 14, 6), (14, 14, 14), (8, 14, 18), (2, 14, 20)) = y_1 (say).

c.n max product {y_1, M} = ((0,0,0), (20, 14, 2), (20, 14, 2), (20, 14, 2), (2, 14, 20)) $\to$ ((10, 14, 4), (20, 14, 2), (20, 14, 2), (20, 14, 2), (2, 14, 20) = y_2 (say).

c.n max product {y_2,M} = ((0, 0, 0), (20, 14, 2), (8, 14, 18), (20, 14, 2), (2, 14, 20)) $\to$ ((10, 14, 4), (20, 14, 2), (8, 14, 18), (20, 14, 2), (2, 14, 20)) = y_3 (say).

c.n max product {y_3, M} = (0, 0, 0), (26, 14, 2), (8, 14, 18), (20, 14, 2), (2, 14, 20)) $\to$ ((10, 14, 4), (20, 14, 2), (8, 14, 18), (20, 14, 2), (2, 14, 20)) = y_4 (say)… I

It is easily verified that $y_4 = y_3$. Thus the resultant neutrosophic triplet groups vector is a fixed point.

For the same x and M we find max min {x, M} = ((0, 0, 0), (2, 14, 20), (14, 14, 14), (8, 14, 18), (8, 14, 18)) $\to$ ((10, 14, 4), (2, 14, 20), (14, 14, 14), (8, 14, 18), (8, 14, 18)) = y_1 (say)

c.n max-min {y_1, M} = ((0,0, 0), (8, 14, 18), (2, 14, 20), (8, 14, 18), (8, 14, 18)) $\to$ ((10, 14, 4), (8, 14, 18), (2, 14, 20), (8, 14, 18), (8, 14, 18) = y_2 (say)

c.n max-min {y_2, M} = ((0, 0, 0), (8, 14, 18), (8, 14, 18), (8, 14, 18), (8, 14, 18)) $\to$ ((10, 14, 4), (8, 14, 18), (8, 14, 18), (8, 14, 18), (8, 14, 18)) = y_3 (say)

c.n max-min {y_3, M} = ((0, 0, 0), (8, 14, 18), (8, 14, 18), (8, 14, 18), (8, 14, 18)) $\to$ ((10, 14, 4), (8, 14, 18), (8,14, 18), (8, 14, 18), (8, 14,18)) = y_4 (say) II

It is clear $y_4 = y_3$ so the resultant is a fixed point which neutrosophic triplet groups row vector.

Clearly I and II are different that is c.n. max-product {x, M} $\neq$ c.n max min {x, M}.

Thus we can have ntgCMs or NTGCMs model using neutrosophic triplet groups when one cannot say the value is totally true or partially true and partiality false but an indeterminacy also exist.

The advantage is this can function akin to SVNs with a change given a true value x the indeterminate or neutral of x and

anti x that is how far x is not true are automatically fixed using the notion of neutrosophic triplet groups concept.

We in view of all these define the notion of a special type of transformation defined as the neutrosophic triplet groups automatic transformation relating. $2Z_{2p}$ with neutrosophic triplet groups of Z_{2p} associated with the neutral element p + 1 first by examples and then make the definition.

Example 4.6. Let S = $\{Z_{106}, \times\}$ be the semigroup under product modulo 106. 106 = 2 × 53 where 53 is the odd prime.

Consider the neutral element 54 of S we see the neutrosophic triplet groups associated with 54 are

{(2,54, 80), (80, 54, 2), (4, 54, 40), (40, 54, 4), (8, 54, 20), (20, 54, 8), (16, 54, 10), (10, 54, 16), (32, 54, 58), (58, 54, 32), (64, 54,82), (82, 54, 64), (22, 54, 94), (94, 54, 22) and so on} $\cup$ {(0, 0, 0) = B.

Clearly cardinality of B is 54.

Now we define a new type of transformation;

$T_{ntg} : 2Z_{106} \to B$ by

T_{ntg} (x) =(x, 54, anti x) for all $x \in Z_{106} \setminus \{0\}$.

Clearly T_{ntg} is well defined infact a one to one map.

We define T_{ntg} (0) = (0, 0, 0).

Conversely we can define $T_{ntg}^{-1} : B \to 2Z_{106}$ by

T_{ntg}^{-1} {(x, 54, anti x)} = x for all x ≠ 0 and T_{ntg}^{-1} ((0, 0, 0)) = 0.

Clearly T_{ntg}^{-1} is also well defined.

The main purpose for defining the transformation T_{ntg}^{-1} and T_{ntg} is that we want to go from the usual MODCMs model to ntgCMs or NTGCM or NtgCMs model and vice versa under the max product and max min operations. So according to the wishes of the expert we can choose to work with MODCMs or ntgCMs depending on the fact whether the author is interested in taking up the issue of indeterminacy not.

We are yet to fix such special type of transformations for any arbitrary n of $Z_{n.}$

We have defined it only for the case n = 2p, p an odd prime even for n = 3p we have the formula but should know what the sum adds upto. Only knowing the structure is dependent on the form of p of n = 3p.

Here also once the neutral element is fixed given any $x \in 3\ Z_{3p} \setminus \{0\}$ the neutrosophic triplet group (x, neut x, antix) is fixed so the human bias can be totally eliminated by this method.

So one can work with MOD cognitive maps model with entries from $2Z_{2p}$ or $3Z_{3p}$ only and can easily transform the resultant to neutrosophic triplet groups cognitive models.

To the effect we will supply one example.

Example 4.7. Let C_1, C_2, C_3 and C_4 be four concepts / nodes associated with some problem. The edge weights are taken from $2Z_{26} = \{0, 2, 4, 6, 8, 10, \ldots, 22, 24\} \subseteq Z_{26}$.

Let G be the directed MOD graph with vertices C_1, C_2, C_3 and C_4 and edge weights from $2Z_{26}$ be given by the following Figure 4.7.

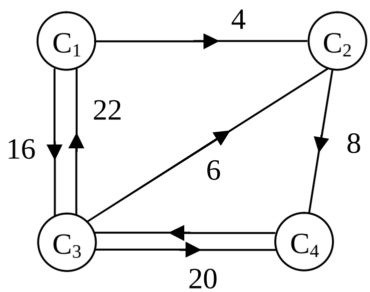

Figure 4.7

Let M be the connection matrix associated with the Figure 4.7 given in the following.

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} 0 & 4 & 16 & 0 \\ 0 & 0 & 0 & 8 \\ 22 & 6 & 0 & 20 \\ 0 & 0 & 10 & 0 \end{bmatrix} \end{array}$$

Let $X = \{(a_1, a_2, a_3, a_4) / a_i \in 2Z_{26}; 1 \leq i \leq 4\}$ be the MOD row matrix which serves as the state vector of the dynamical system.

We will be using only max product operation and max min operation on $x \in X$ and this M.

Let x = (4, 12, 0, 0) $\in$ X be the initial state vector associated with the dynamical system.

max-product {x, M} = (0, 16, 12, 18) $\to$ (4, 16, 12, 18) y_1 (say) after updating.

We now find max product {y_1, M} = (4, 20, 24, 24) = y_2 (say)

max-product {y_2, M} = (8, 16, 12, 12) = y_3 (say)

max-product {y_3, M} = (4, 20, 24, 24) = y_4 (say)

Clearly $y_2 = y_4$ so the resultant is a MOD limit cycle.

(4, 20, 24, 24) $\to$ (8, 16, 12, 12) $\to$ (4, 20, 24, 24).

Now if we use the same set of x and M and use the max min operation on them.

max-min {x, M} = (0, 4, 4, 8) $\to$ (4, 4, 4, 8) = y_1 (say)

max-min {y_1, M} = (4, 4, 8, 4) = y_2 (say)

max-min {y_2, M} = (8, 6, 4, 8) = y_3 (say)

max-min {y_3, M} = (4, 4, 8, 6) = y_4 (say)

max-min {y_4, M} = {8, 6, 6, 8} = y_5 (say)

max-min {y_5, M} = (6, 6, 8, 6) = y_6 (say)

max-min {y_6, M} = (8, 6, 6, 8) = y_7 (say).

Clearly $y_7 = y_5$ so the MOD resultant vector is a limit cycle given by (8, 6, 6, 8) $\to$ (6, 6, 8, 6) $\to$ (8, 6, 6, 8).

Now we use the transformation for each graph matrix and initial state vectors and arrive at the result.

Now the MOD graph G using the T_{ntg} transformation for the edge weights of G is transformed to T_{ntg} (G).

We first find the neutral elements of Z_{26}. The neutral elements are 13 and 14, of course 13 does not contribute to nontrivial triplet groups other than (0, 0, 0) and (13, 13, 13).

The neutrosophic triplet groups associated with the neutral element 14 are

B = {(14, 14, 14), (2, 14, 20), (20, 14, 2), (4, 14, 10), (10, 14, 4), (8, 14, 18), (18, 14, 8), (16, 14, 22), (22, 14, 16), (6,14, 24), (24, 14, 6), (12, 14, 12)}. We adjoin (0, 0, 0) with this collection B.

Now T_{ntg} (0) = (0, 0, 0), T_{ntg} (2) = (2, 14, 20),

T_{ntg} (4) = (4, 14, 10), T_{ntg} = (6) = (6, 14, 24)

T_{ntg} (8) = (8, 14, 18), T_{ntg} (10) = (10, 14, 4)

T_{ntg} (12) = (12, 14, 12), T_{ntg} (14) = (14,14, 14)

T_{ntg}(16) = (16, 14, 22), T_{ntg} (18) = (18, 14, 8)

T_{ntg} (20) = (20, 14, 2), T_{ntg} (22) = (22, 14, 16) and

T_{ntg} (24) = (24, 14, 6).

Now using this special type of transformation we transform the MOD graph G into T_{ntg} (G) which is given in the following Figure 4.8.

T_{ntg} (G) =

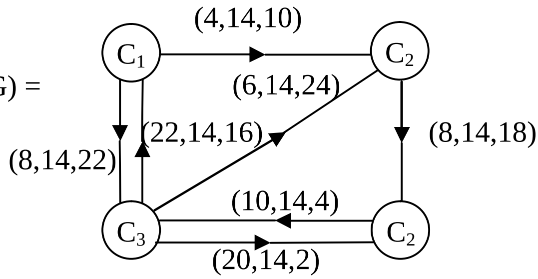

Figure 4.8

It is to be noted that only the edge weights gets transformed. We see no changes in the vertices they remain as C_1, C_2, C_3 and C_4.

However the MOD edge weights is transformed by T_{ntg} into neutrosophic triplet groups.

This is shown in Figure 4.8.

Now it is pertinent to keep on record that we can transform a appropriate MOD graph into neutrosophic triplet groups graph using the transformation T_{ntg} which is shown in Figure 4.8.

Now every MOD matrix can be transformed by T_{ntg} into a neutrosophic triplet groups matrix, that is if M = (m_{ij}) then T_{ntg} (M) = T_{ntg} (m_{ij}) = $(T_{ntg}$ $(m_{ij}))$.

Now we will show how the 4 × 4 MOD matrix M given in this problem is transformed by T_{ntg}.

$$T_{ntg}(M) = \begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline C_1 & (0,0,0) & (4,14,10) & (16,14,22) & (0,0,0) \\ C_2 & (0,0,0) & (0,0,0) & (0,0,0) & (8,14,18) \\ C_3 & (22,14,16) & (6,14,24) & (0,0,0) & (20,14,2) \\ C_4 & (0,0,0) & (0,0,0) & (10,14,4) & (0,0,0) \end{array}$$

$T_{ntg}(M)$ is the transformed MOD matrix into the neutrosophic triplet group matrix.

We will now find the effect of $T_{ntg}(x)$ on T_{ntg} (M) using c.n max product operation and c.n max min operation.

However we can show the method of max product and max min of {x, M} yield a MOD resultant can be transformed into neutrosophic triplet groups vector by the special transformation T_{ntg}.

We first find c.n max-prod {x, M} using x = ((4, 14, 10), (12, 14, 12), (0, 0, 0), (0, 0, 0)).

c.n max-product {T_{ntg} (x), T_{ntg} (M)} = ((0, 0, 0), (16, 14, 22), (12, 14, 12), (18, 14, 8)) $\rightarrow$ ((4, 14, 10), (16, 14, 22), (12, 14, 12), (18, 14, 18)) = y_1 (say) (after updating)

(It is important to note at this juncture $T_{ntg}^{-1}(y_1)$ = (4, 16, 12, 18) which is the value we obtained using the operation max product {x, M}).

We find c.n max product {y_1, $T_{ntg}(M)$} = ((4, 14, 10), (20, 14, 2), (24, 14, 6), (24, 14, 6)) = y_2 (say)

c.n max product {y_2, $T_{ntg}(M)$} = ((8, 14, 18), (16, 14, 22), (12, 14, 12), (12, 14, 12)) = y_3 (say).

c.n max product $\{y_3, M\}$ = ((4, 14, 10), (20, 14, 2), (24, 14, 6), (24, 14, 6)) = y_4 (say).

We see $y_2 = y_4$ thus the neutrosophic triplet groups resultant vector is a limit cycle given by ((4, 14, 10), (20, 14, 2), (24, 12, 6), (24, 14, 6)) $\rightarrow$ ((8, 14, 18), (16, 14, 22), (12, 14, 12), (12, 14, 12)) $\rightarrow$ ((4, 14, 10), (20, 14, 2), (24, 14, 6), (24, 14, 6)).

Now for the same T_{ntg} (x) and T_{ntg} (M) we obtain the

c.n max-min $\{T_{ntg}(x), T_{ntg}(M)\}$ = ((0, 0, 0), (4, 14, 10), (4, 14, 10), (8, 14,18))

$\rightarrow$ ((4, 14, 10), (4, 14, 10), (4, 14,10), (8, 14 18)) = y_1 (say).

c.n max min $\{y_1\ T_{ntg}\ (M)\}$ = ((4, 14, 10), (4, 14, 10), (8, 14, 18), (4, 14, 10)) = y_2 (say)

c.n max min $\{y_2, T_{ntg}\ (M)\}$ = ((8, 14, 18), (6, 12, 24), (4, 14, 10), (8, 14, 18)) = y_3 (say)

c.n max min $\{y_3, T_{ntg}(M)\}$ = ((4, 14, 10), (4, 14, 10), (8, 14, 18), (6, 14, 24)) = y_4 (say)

c. n max min $\{y_4, T_{ntg}$ (M) = ((8, 14, 18), (6, 14, 24), (6, 14, 24), (8, 14, 18)) = y_5 (say)

c.n max min $\{y_5, T_{ntg}\ (M)\}$ =((6, 14, 24), (6, 14, 24), (8, 14, 18), (6, 14, 24)) = y_6 (say)

c.n max min $\{y_6, T_{ntg}\ (M)\}$ =((8, 14, 18), (6, 14, 24), (6, 14, 24), (8, 14, 18)) = y_7 (say).

It is clearly $y_5 = y_7$ is a limit cycle given by

((8, 14, 18), (6, 14, 24), (6, 14, 24), (8, 14, 18)) $\to$ ((6, 14, 24), (6, 14, 24), (8, 1,4 18), (6, 14, 24)) $\to$ ((8, 14, 18), (6, 14, 24), (6, 14, 24), (8, 14, 18)).

We see T_{ntg}^{-1} $((y_5))$ = (8, 6, 6, 8).

It is pertinent to observe that the limit cycle of max product (x, M) is c.n max product $(T_{ntg}(x), T_{ntg}(M))$ and vice versa. So according to convenience one can work with either MOD max product or c.n max product and convert from one another using the special transformation $T_{ntg}(x)$ or $T_{ntg}^{-1}(x)$.

Next we proceed onto describe neutrosophic triplet groups relational maps (ntgRMs) model. It is pertinent to keep on record MOD relational maps model was developed, defined and described in [21]. Here we define and describe the neutrosophic triplet groups relational maps (NTGRMs or NtgRMs) model.

Let S = $\{Z_{2p}, \times\}$ be semigroup under product modulo 2p.

B = {2, 4, …, (p – 1)2} be the cyclic group of order p – 1 with p + 1 as the identity.

Let D $\cup$ {(0, 0, 0)} = {collection of all neutrosophic triplet groups associated with the neutral element p +1} $\cup$ {(0, 0, 0)}.

Now if we have a bipartite graph G with edge weights from D then we define G to be a neutrosophic triplet groups bipartite graph which takes edge weights from D.

We will describe this by some examples.

Example 4.8. Let S = $\{Z_{38}, \times\}$ be the semigroup under product modulo 38. The neutral elements of Z_{38} are 19 and 20.

19 does not yield any nontrivial neutrosophic triplet groups. Only the neutral 20 contributes to nontrivial neutrosophic triplet groups given by

A = {(20, 20, 20), (2, 20, 10), (10, 20, 2), (4, 20, 24), (24, 20, 4), (8, 20, 12), (12,20, 8), (16, 20, 6), (6, 20, 16), (32, 20, 22), (22, 20, 32), (14, 20, 14), (28, 20, 26), (26, 20, 28), (18, 20, 18), (30, 20, 30), (34, 20, 34), (36, 20, 36)}. They are 18 in number we define A $\cup$ {(0, 0,0)} = B.

We will be using values this B as edge weights and obtain the bipartite graphs.

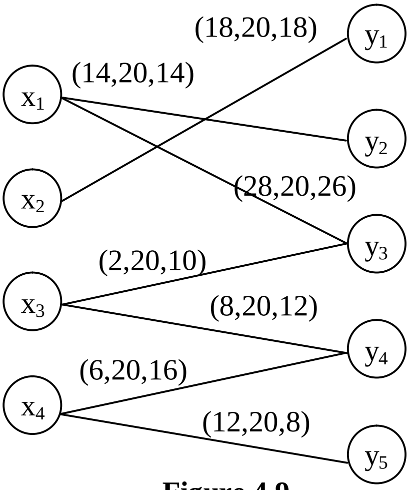

Figure 4.9

Clearly G is a neutrosophic triplet groups bipartite graph with edge weights from B.

The relational matrix R which is also known as the neutrosophic triplet groups relational matrix associated with G is as follows.

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & y_3 & y_5 & y_6 \end{array} \\ \left[\begin{array}{ccccc} (0,0,0) & (14,20,14) & (28,20,26) & (0,0,0) & (0,0,0) \\ (18,20,18) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (2,20,10) & (8,20,12) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (6,20,16) & (12,20,8) \end{array}\right] \end{array}$$

We can perform only two types of operations using R.

c.n max-product {x, R} or c.n max min {x, R} where

$x \in X = \{(a_1, a_2, a_3, a_4) / a_i \in A, 1 \leq i \leq 4\}$ and

$y \in Y = \{(b_1, b_2, b_3, b_4, b_5) /b_i \in A; 1 \leq i \leq 5\}$.

We will describe this situation in the following

Let x = ((2, 20, 10), (6, 20, 16), (36, 20, 36), (0, 0, 0)) $\in$ X.

We find c.n max-product (x, R).

c.n max-product (x, R) = ((32, 24, 22), (28, 20, 26), (18, 20, 18), (34, 20, 34), (0, 0, 0)) = y_1 (say)

c.n max-product ({ y_1^t, R} = (y, R^t))

((12, 20, 8), (6, 20, 16), (36, 20, 36), (14, 20, 14)) = x_1 (say)

c.n max-product {x_1, R} = ((32, 20, 22), (16, 20, 6), (34, 20, 34), (22, 20, 32), (16, 20, 6)) = y_2 (say)

c.n max-product ({ y_2^t, R} = (y_2R^t)) = ((34, 20, 34), (6, 20, 16), (30, 20, 30), (18, 20,18)) = x_2 (say).

c.n max-product {x_2, R} = ((32, 20, 22), (30, 20, 30), (22, 20, 32), (32, 20, 22) (26, 20, 28)) = y_2 (say).

We find c.n max-product {y_2 R^t} = ((8, 20, 12), (6, 20, 16), (28, 20, 26), (8, 20, 12)) = x_3 (say)

c.n max-product {x_3, R} = ((32, 20, 22), (36, 20, 36), (34, 20, 34), (34, 20, 34), (20, 20, 20)) = y_3 (say).

c.n max-product {y_3, R^t} = ((10, 20, 2), (30, 20, 30), (30, 20, 30), (14, 20, 14)) = x_4 (say)

c.n max-product {x_4, R} = ((8, 20, 12), (26, 20, 28), (22, 20, 32), (12, 20,8), (16, 20, 6)) = y_4 (say).

c.n max-product {y_4, R^t) = ((16, 20, 6), (30, 20, 30), (20, 20, 20), (34, 20, 34)) = x_5 (say)

c.n max-product (x_5, R) = ((8, 20, 12), (34, 20, 34), (30,20, 30), (14, 20, 14), (28, 20, 26)) = y_5 (say)

We find c.n max-product ((y_5, R^t) = (y_5^t, R)) = ((18, 20, 18), (30,20, 30), (36, 20, 36), (32, 20, 22)) = x_6 (say)

c.n max-product $\{x_6, R\}$ = ((8, 20, 12), (24, 20, 4), (16, 20, 6), (12, 20, 8), (4, 20, 24)) = y_6 (say)

c.n max-product $\{y_6, R^t\}$ = ((32, 20, 22), (30, 20, 30), (32, 20, 22), (34, 20, 34)) = x_7 (say)

c.n max-product $\{x_7, R\}$ = ((8, 20, 12), (30, 20, 30), (26, 20, 28), (28, 20, 26), (28, 20, 26)) = y_8 (say)

c.n max-product $\{y_8, R^t\}$ = ((2, 20, 10), (30, 20, 30), (34, 20, 34), (32, 20, 22)) = x_8 (say)

c.n max-product $\{x_8, R\}$ = ((16, 20, 6), (26, 20, 28), (30, 20, 30), (10, 20, 2), (4, 20, 24)) = y_9 (say)

We find c.n max-product $(y_9\ R^t\}$ = ((28, 20, 26),(22, 20, 32), (22, 20, 32), (22, 20, 32)) = x_9 (say)

c.n max-product $\{x_9, R\}$ = ((16, 20, 6), (12, 20, 8), (24, 20, 4), (24, 20, 4), (36, 20, 36)) = y_{10} (say)

c.n. max-product $\{y_{10}, R^t\}$ = ((26, 20, 28), (22, 20, 32), (10, 20, 2), (30, 20, 30)) = x_{10} (say).

The reader is left with the task of finding the resultant neutrosophic triplet groups state vectors pair. This will end either in a fixed point pair or a limit cycle pair as the dynamical system is built only on finite collection of neutrosophic triplet groups.

Thus we see the neutrosophic triplet groups relational maps model under max product can be used in need especially when the concept of indeterminacy is also present.

Now for the some $x \in X$ and for this R we use. The c.n max-min operation and determine the neutrosophic triplet groups resultant vector pair which may be a fixed point pair or a limit cycle pair in the following.

Let x = ((4, 20, 24), (2, 20, 10), (0, 0, 0), (0, 0, 0)) $\in$ X.

c.n max min {x, R} = ((2, 20, 10), (4, 20, 14), (4, 20, 14), (0, 0, 0), (0, 0, 0)) = y(say)

c.n max min $\{y_1\ R^t\}$ = ((4, 20, 14), (2, 20, 10), (4, 20, 14), (0, 0, 0) = x_1 (say)

c.n max min $\{x_1, R\}$ = ((2, 20, 10), (4, 20, 14), (2, 20, 10), (4, 20, 14), (0, 0, 0)) = y_2 (say)

c.n max min $\{y_2, R^t\}$ = ((4, 20, 14), (2, 20, 10), (4, 20, 14), (4, 20, 14)) = x_2 (say)

c.n max min $\{x_2, R\}$ = ((2, 20, 10), (4, 20, 14), (2, 20, 10), (4, 20, 14), (4, 20, 14)) = y_3 (say)

c.n max min $\{y_3, R^t\}$ = ((4, 20, 14), (2, 20, 10), (4, 20, 14), (4, 20, 14)) = x_3 (say)

It is clear $x_2 = x_3$ so we get the neutrosophic triplet groups resultant as a fixed pair given by

{((4, 20, 14), (2, 20, 10), (4, 20, 14), (4, 20, 14)), ((2, 20, 10), (4, 20, 14), (2, 20, 10), (4, 20, 14), (4, 20, 14))}.

We suggest the following open problem.

Suppose we have a $m \times n$ neutrosophic triplet group relational matrix R with entries from neutral element p + 1 of $Z_{2p.}$

Let $X = \{(a_1, a_2, \ldots, a_m) / a_i \in$ {collection of neutrosophic triplet group of the neutral element p + 1}; $1 \leq i \leq m\}$ and

$Y = \{(b_1, b_2, \ldots, b_n) / b_i \in$ {collection of all neutrosophic triplet groups associated with the neutral element p+1}; $1 \leq i \leq n\}$.

Can we say c.n max min {x, R} converges to a limit cycle pair or a fixed point pair faster than the operator c.n. max product {x, R} in general for $x \in X$ and the given R?

Obtain any other special features enjoyed by this model.

Now we suggest how this can be used to build algebraic codes. However some information in this regard has been described and defined in chapter II of this book.

We only restrain ourselves to the semigroup $\{Z_{2p}, \times\}$ where p is an odd prime.

B = {(0, 0, 0) and the collection of all neutrosophic triplet groups associated with the neutral element p + 1}.

We cannot define + operation B.

B is compatible with respect to only product modulo 2p, min operation and max binary operation where we use the special type of ordering called face value ordering which in general is not compatible with sum or product operations.

So when building the notion of parity check matrix or generator matrix we do not have the notion of linear independence either of the row elements of the matrix or the column elements of the matrix. However the algebraic codes built using the elements of B happens to be similar to the usual or classical codes built using finite fields Z_p.

We have the limitations for B cannot be converted into the classical field finite order as it is impossible to define the operation of addition as the set B is not closed under + ; that is if $x, y \in B$ clearly $x + y \notin B$ for every or any $x, y \in B$.

We have discussed elaborately about these codes in chapter II.

Further {B, max, min} and {B, max, product} is only a semiring or a semifield. It is impossible to define or make $\{B, + \times\}$ into a finite field.

With these limitations we build the algebraic codes of neutrosophic triplet groups.

Finally it is pertinent to keep on record that throughout our discussion we restrain and in this book only $\{Z_{2p}, \times\}$ is used.

We will illustrate by examples.

Example 4.9. Let $S = \{Z_{26}, \times\}$ be the semigroup under product modulo 26. The neutral elements of S are 13 and 14.

The neutrosophic triplet groups associated with 14 are

B = {(14, 14, 14), (2, 14, 20), (20, 14, 2), (4, 14, 10), (10, 14, 4), (8, 14, 18), (18, 14, 8), (16, 14, 22), (22, 14, 16), (6, 14, 24), (24, 14, 6), (12, 14, 12)}.

A = B ∪ {(0, 0, 0)}.

We now build a (7, 3) neutrosophic triplet group code using the neutrosophic triplet group generator matrix G where

$$G = \begin{bmatrix} (6,14,24) & (0,0,0) & (12,14,12) & (4,14,10) & (0,0,0) & (0,0,0) & (20,14,2) \\ (0,0,0) & (10,14,4) & (0,0,0) & (0,0,0) & (2,14,20) & (0,0,0) & (0,0,0) \\ (2,14,20) & (0,0,0) & (0,0,0) & (14,14,14) & (0,0,0) & (16,14,22) & (0,0,0) \end{bmatrix}.$$

We see x = ((2, 14, 20), (8, 14, 18), (2, 14, 20))

max-min {x, G} = ((12, 14, 12), (2, 14, 20), (24, 14, 6), (8, 14, 18), (16, 14, 22), (6, 14, 24), (14, 14, 14))

So x = ((2, 14, 20), (8, 14, 18), (2, 14, 20)) generates the code ((12, 14, 12), (2, 14, 20), (24, 14, 6), (8, 14, 18), (16, 14, 22), (6, 14, 24),(14, 14,14))

This is the way we arrive at the neutrosophic triplet group code word.

Now we define the Hamming distance between two neutrosophic triplet groups state vectors as the number of places in which they differ as triplets.

We will illustrate this by some examples.

Let X = {(a_1, a_2, a_3, a_4) / $a_i \in$ A, 1 ≤ i ≤ 4} be the neutrosophic triplet groups row vector.

Let x = ((2, 14, 20), (10, 14, 4), (8, 14, 8), (16, 14, 22)) and y = ((10, 0, 0), (14, 14, 14), (8, 14, 8), (16, 14, 22)) ∈ X.

The Hamming distance between x and y denoted by d_{ntg} (x, y) = 2; that is the first and the second entries of x and y are different where as the 3rd and 4th coordinates of x and y are the same.

Now we are not in a position to apply coset leaders method as we cannot define sum of two vector, however if we define min operation then we will always have the coset leader to the needed vector which cannot be practical but we need to see whether product can be defined for max operation will have no effect on it as it will between so no correction can be made.

Hence we have to try for the some other method this cannot be easily achieved so we have to seek after some other method to find error correction but error detection can be achieved by the Hamming distance method.

Now we try to define the algebra neutrosophic triplet groups code in the standard form. First we enlist the short comings.

The identity with respect to product is (p + 1, p + 1, p + 1), but this is not the identity with respect max or min for in case of max or min we cannot have identity except in that case largest element and least element respectively will serve the purpose.

Hence we use only max product and max min we see the largest element will be chosen.

Suppose we have a parity check matrix say

$$H = \begin{bmatrix} (2,14,20) & (0,0,0) & (0,0,0) & (14,14,14) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (4,14,10) & (2,14,20) & (0,0,0) & (14,14,14) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (16,14,22) & (0,0,0) & (0,0,0) & (14,14,14) & (0,0,0) \\ (20,14,2) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (14,14,14) \end{bmatrix}$$

Where H is the neutrosophic triplet group matrix.

Let us construct the G using this H.

$$G = \begin{bmatrix} (2,14,20) & (0,0,0) & (0,0,0) & (20,14,2) & (14,14,14) & (0,0,0) & (0,0,0) \\ (0,0,0) & (4,14,10) & (0,0,0) & (0,0,0) & (0,0,0) & (14,14,14) & (0,0,0) \\ (0,0,0) & (0,0,0) & (16,14,22) & (0,0,0) & (0,0,0) & (0,0,0) & (14,14,14) \end{bmatrix}$$

We now calculate the $G \times H^t$ using first c.n max product then we use the c.n max min and find out GH^t.

c.n max-product $\{G, H^t\}$ =

$$\begin{bmatrix} (2,14,20) & (0,0,0) & (0,0,0) & (14,14,14) \\ (0,0,0) & (4,14,10) & (2,14,20) & (0,0,0) \\ (0,0,0) & (0,0,0) & (16,14,22) & (0,0,0) \\ (20,14,2) & (0,0,0) & (0,0,0) & (0,0,0) \end{bmatrix}$$

We see c.n max min (G, H^t) =

$$\begin{bmatrix} (14,14,14) & (14,14,14) & (0,0,0) & (0,0,0) \\ (0,0,0) & (4,14,10) & (14,14,14) & (0,0,0) \\ (0,0,0) & (2,14,20) & (16,14,22) & (14,14,14) \end{bmatrix}$$

We see both yield different matrices so we are sure in general both the c.n max min and c.n max product operations take different values so while working with them appropriate modifications ought to be used in the place of need.

One can define with appropriate modifications the notion of Hamming code, Parity check code etc. The main short coming in developing these codes is under c.n max product and c.n min product we do not have the concept of identity barring the domination of the largest element and the least element respectively.

Next we proceed onto show how these collection of neutrosophic triplet groups can be given a semiring structure. Infact they are finite semifields.

Now one is in a position to solve the open problems proposed in giving examples of finite semifields under the operation max, product or max, min of order p, p any odd prime.

Infact these semirings which are semifields can be constructed to any prime order and can infact be generated by product operation by a single elements.

We can this semirings can be given also the ring structure in a very queer way.

B = {collection of neutrosophic triplet groups associated with the neutral element p + 1 of Z_{2p}, p an odd prime} $\cup$ {(0, 0, 0)}, {B, $\times$} is a group barring {(0, 0, 0)} and {B, max} is a semigroup.

So {B, $\times$, max} is a ring we call it in a queer way for while defining product B \ {(0, 0, 0)} is a group under $\times$. If {(0, 0, 0)} is taken with B, {B, $\times$} is only a semigroup. So B is only a semi field under {max-product} or {max min}.

Once again we wish to keep on record that when we define max {(a, neut(a), anti(a), (b, neut(b)), anti(b))} = (max {a, b}, neut (max {a, b}), anti (max {a, b)).

This order defined as face value ordering which is not compatible under product likewise min {(a, neut(a), anti(a)), (b, neut(b), anti(b)} = (min {a, b}, neut (min {a,b}), anti (min {a, b})), min {a, b} and max {a, b} are found face compatible under ordering.

If 20 and 28 $\in$ $2Z_p$ \ {0} by face value ordering max value {20, 28} = 28 and min {20, 28} = 20.

This form of study has already been discussed [20-21].

So {B, max product} is a finite semifield with p number of elements in it.

Interested reader can find special type of coding theory using these semifields.

Are these lattices under max, min operation? We will test whether first the form semi lattices under product.

We see under both max or min operation they form a totally ordered set hence will be a chain lattice. In case of this B we see using max min we get a chain lattice of order 13 given by the following Figure 4.10.

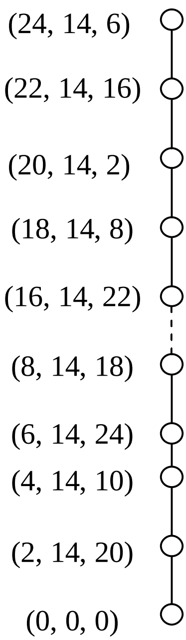

Figure 4.10

However product cannot be defined on B to yield a lattice or semilattice. Only B \ {(0, 0, 0)} is a cyclic group of order 12.

We first give examples of the group of neutrosophic triplet groups of graphs.

Example 4.10. Let S = $\{Z_{14}, \times\}$ be the semigroup under product modulo 14.

The neutral elements of Z_{14} are 7 and 8.

7 contributes only for trivial neutrosophic triplet groups.

8 alone contributes for non trivial neutrosophic triplet groups given by the following set B.

B = {(8, 8, 8), (2, 8, 4), (6, 8, 6), (4, 8, 2), (10, 8, 10), (12, 8, 12)}; B is clearly a cyclic group with (8, 8, 8) as its multiplicative identity.

We now give the graph of B in the following.

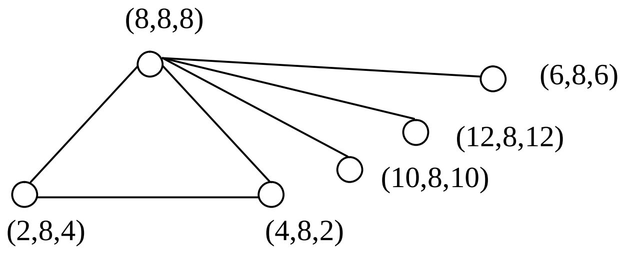

Figure 4.11

We will give yet another example.

Example 4.11 Let S = {Z_{26}, ×} be the semigroup under product modulo 26. The neutral elements of Z_{26} are 13 and 14. 13 contributes only for neutrosophic triplet group.

The neutrosophic triplet groups associated with 14 are

B = {(14, 14, 14), (2, 14, 20), (20, 14, 2), (4, 14, 10), (10, 14, 14), (8, 4, 18), (18, 4, 8), (16, 14, 22), (22, 14, 16), (6, 14, 24), (24, 14, 6), (12, 14, 12)}.

Clearly B is a cyclic group of order 12 with (14, 14, 14) as the identity.

Now we give the neutrosophic triplet group graph associated with B the classical group of neutrosophic triplet groups in the following.

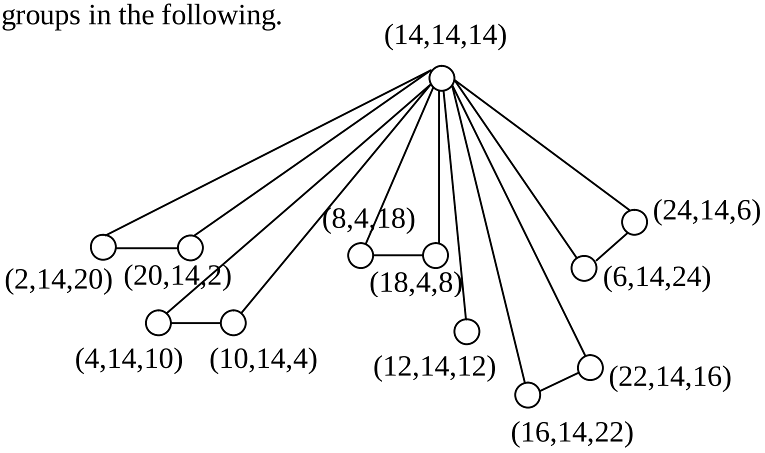

Figure 4.12

We can say by looking at the figure there is one elements of order two where are 5 pairs of elements which are inverses of each other.

Example 4.12. Let S ={Z_{39}, ×} be the semigroup under product. The neutrals of Z_{39} are 13 and 27. However the neutrosophic triplet groups associated with 13 are trivial. So we have to work only with the neutral element 27. The neutrosophic tripletgroups associated with 27 are

B = {(27, 27, 27), (3, 27, 9), (9, 27, 3), (6, 27, 24), (24, 27, 6), (18, 27, 21), (21, 27, 18), (15, 27, 33), (33, 27, 15), (12, 27, 12), (30, 27, 36), (36, 27, 30)}.

Clearly B is a cyclic group neutrosophic triplet groups of order 12. We now give neutrosophic triplet group graph of the group B as follows.

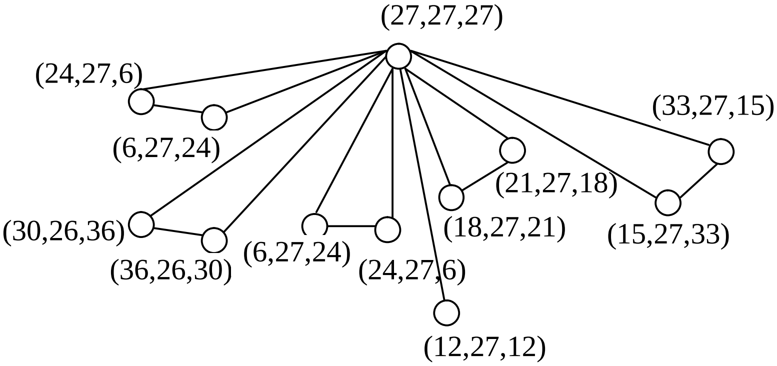

Figure 4.13

We see the two groups of neutrosophic triplet groups graph are the same so it goes without any saying both are just isomorphic however one is a group associated with Z_{26} and the other with Z_{30}.

Now we proceed onto study the classical group of neutrosophic triplet groups associated with Z_{65} where 65 = 13 × 5.

Example 4.13. Let S = {Z_{65}, ×} be the semigroup under product modulo 65. Clearly 26 is a neutral element of Z_{65}. 40 is another neutral element of Z_{65}. The neutral element 26 gives four neutrosophic triplet groups given by

A ={(26, 26, 26), (13, 26, 52), (52, 26, 13), (39, 26, 39)} is a classical group of neutrosophic triplet groups of order four. A has the following neutrosophic triplet groups graph.

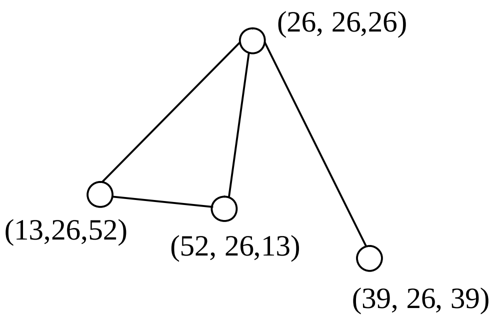

Figure 4.14

Now we enumerate the neutrosophic triplet groups associated with the neutral element 40.

B = {(40, 40, 40), (5, 40, 60), (60,40,5), (25, 40, 25), (10, 40, 30), (30, 40, 10), (50, 40, 45), (45, 40, 50), (20, 40, 15), (15, 40, 20), (35, 40, 55), (55, 40, 35)}.

The graph of the classical group of neutrosophic triplet graphs is as follows.

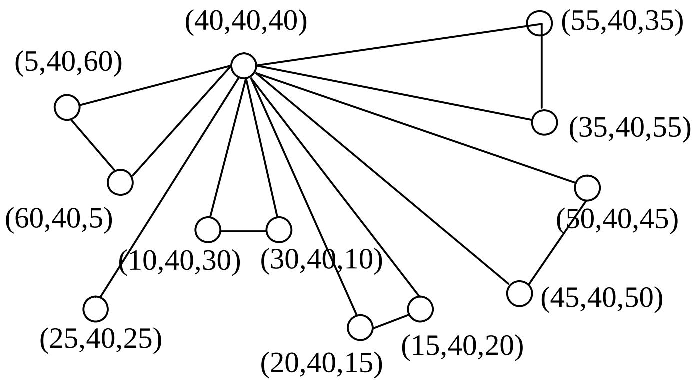

Figure 4.15

In view of all these we make the following conjecture.

Conjecture 4.2. Let $\{Z_{pq}, \times\}$ be the semigroup under product modulo pq, p and q two distinct prime

i) For all primes p < q (p = 2, 3, 5, … and p < q) we see there is a neutral element which is a multiple of p such that associated with p there are q – 1 number of neutrosophic triplet groups which forms a classical group and the graphs of all these groups for varying are the same for any fixed q such that p < q.

We see in case of $\{Z_{pq}, \times\}$ with q = 13 all neutrosophic triplets groups associated with tp; t > 0 for a classical group of order (q – 1) and the graphs of all these groups has only 12 elements and all of them have the same graph verified for p = 2, 3 and 5 in this book. The reader is left with the task of finding when p = 7 and 11.

It is pertinent to keep on record that for any S = $\{Z_{pq}, \times\}$ distinct where p and q are two distinct primes we have two distinct neutrals and they contribute neutrosophic triplet groups collection of orders p – 1 and q = 1 and they are or classical groups with a × b = identity or a^2 = identity.

That is they enjoy only this special feature as they are cyclic groups of even order.

We will briefly discuss about the neutral elements and the neutrosophic groups of Z_{pqr}, p, q and r are three distinct primes.

Let S = $\{Z_{105}, \times\}$ be the semigroup under product. The neutrals of Z_{105} are 15, 21, 36, 70, 85 and 91.

The neutrosophic triplet groups associated with 36 are \

B = {(36, 36, 36), (3, 36, 12), (12, 36, 3), (9, 36, 39), (39, 36, 9), (27, 36, 48), (48, 36, 27), (81, 36, 51), (51, 36, 81), (33, 36, 87), (87, 36, 33), (99, 36, 99), (6, 36, 6), (18, 36, 72), (72, 36, 18), (54, 36, 24), (24, 36, 54), (57, 36, 78), (78, 36, 57), (66, 36, 96), (96, 36, 66), (93, 36, 102), (102, 36, 93), (69, 36, 69) and on.

The reader is expected to find out all those neutral elements which only contribute to neutrosophic triplet groups which are trivial.

We just define the graphs whose groups are such that either x × y = identity or a^2 = identity as line-triangle centered graphs. We record at this juncture all classical graph neutrosophic triplet groups graphs are only triangle line centered graphs.

The only open problem we propose is that can when will (that is for what values of p and q of Z_{pq}, p and q two distinct primes); the line-triangle graph have only one line and rest triangles?

When will they have more than one line in the line-triangle graphs of these classical group of neutrosophic triplet groups.

Problems

1. Let S = $\{Z_{158}, \times\}$ be the semigroup under product.

   i) Prove 80 is the neutral element of Z_{158}.

   ii) Prove B = {Neutrosophic triplet groups collection associated with the neutral elements 158} $\cup$ {(0, 0, 0)} is a semigroup.

   iii) Let $A = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle/ a_i \in B;\ 1 \leq i \leq 16 \right\}$ be the collection of all 4×4 matrices of neutrosophic triplet groups.

   $X = \{(x_1, x_2, x_3, x_4) / x_i \in B;\ 1 \leq i \leq 4\}$ be the neutrosophic triplet groups of row matrices.

   a) Find the number of fixed points of c.n max-product = {x, M}; $x \in X$ and $M \in A$.

   b) Find the number of fixed points of c.n. max min {x, M} for $x \in X$ and $M \in A$

c) How many limit cycles exist in
c.n max product {x, M}, $x \in X$ and $M \in A$?

d) How many limit cycles exists in
c.n max min {x, M} for $x \in X$ and $M \in A$?

e) Prove usual xM is not possible?

f) Find the maximum number of iterations required to arrive at a fixed point or limit cycle using c.n max product {x, M}, $x \in X$, $M \in A$.

g) Find the maximum number of iterations required to arrive at a fixed point or limit cycle using c.n max min {x, M}, $x \in X$ and $M \in A$.

h) Does max product {x, M} takes more iterations or max min {x, M} takes more iterations in general for $x \in X$ and $M \in A$?

i) How does the neutrosophic triplet group differ from usual MOD data?

2. Let $S = \{Z_{279}, \times\}$ be the semigroup under product modulo 279.

i) Enumerate the neutral elements of Z_{279}.

ii) Which of the neutral elements contribute to nontrivial cyclic groups of neutrosophic triplet groups or atleast a classical group B.

iii) Let A = {collection of all 5×5 matrices of neutrosophic triplet groups using $B \cup \{(0, 0, 0)\}$

Let X = $\{(a_1, a_2, a_3, a_4, a_5) / a_i \in B \cup \{(0, 0, 0)\}$ $1 \leq i \leq 5\}$ be the row matrix of neutrosophic triplet groups.

Study questions (a) to (g) of (iii) of problem (1) for this Q and $\times$.

3. Let S = $\{Z_{202}, \times\}$ be the semigroup under product modulo 202.

A = {collection of all 6×6 matrices with entries from $2Z_{202}$} be the collection of MOD matrices.

Let X = $\{(a_1, a_2, \ldots, a_6) / a_i \in 2Z_{202}, 1 \leq i \leq 6\}$ be the collection of MOD row matrices.

i) Using the $X \circ M$ operation find the fixed point or limit cycle of the system for $x \in X$ and m $\in$ A.

ii) Find the maximum number of iterations required to arrive at a fixed point or limit cycle of xM for the particular $x \in X$ and $M \in A$.

iii) Use the max-product {x,M} and find the maximum number of iterations required to arrive at a fixed point or a limit cycle.

iv) Can we say in general xM takes more number of iterations to arrive at a fixed point than max product {x, M} for all $x \in X$ and $M \in A$.

v) Using max min {x, M} and test for the maximum number of iterations to arrive at a fixed point or a limit cycle for $x \in X$ and $M \in A$.

vi) Which of xM or max min {x, M} will need in general more number of iterations to arrive at a fixed point for $x \in X$ and $M \in A$?

vii) Which of max min {x, M} or max product {x, M} in general need more number of iterations to arrive at a fixed point or a limit cycle?

viii) Compare all the three operations max product, max min and $x \circ M$ and test out which of the operations is sensitive or each is suitable depending on the problem on which they are applied.

4. Let $P = \{Z_{309}, \times\}$ be the semigroup under product modulo 309.

   B = {collection of all 7 × 7 matrices with entries from $3Z_{309}$ be the MOD matrix collection.

   $X = \{(a_1, a_2, \ldots, a_7) / a_i \in 3Z_{309}, 1 \leq i \leq 7\}$ be the MOD row matrix collection.

   Study questions (i) to (viii) of problem (3) for this P, B and X.

5. Let $S = \{Z_{106}, \times\}$ be the semigroup under product modulo 106.

i) Prove 54 and 53 are the only neutrals of Z_{106}.

ii) Prove 53 can only contribute to trivial neutrosophic triplet group collection.

iii) Find the classical group of neutrosophic triplet groups associated with the neutral element.

iv) Let $A = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \end{array}$ be the dynamical system associated with the ntg. Relational (ntgRMs or NtgRMs or NTGRMs) maps $a_i \in$ {collection of all neutrosophic triplet groups associated with the neutral element 54} $\cup$ {(0, 0 0)} = B; $1 \leq i \leq 21$.

a) Draw the neutrosophic triplet group bipartite graph by giving some values to $a_i \in B$.

b) Transform using T_{ntg}^{-1} to get the MOD relational maps matrix.

c) Let Y = {(d_1, d_2, d_3, … d_7} / $d_i \in B$; $1 \leq i \leq 7$} and X ={(a_1, a_2, a_3) / $a_i \in$ B; $1 \leq i \leq 3$} be the neutrosophic triplet groups row matrices.

d) Using some fixed values of x (or y) find the fixed point or limit cycle of the system.

6. Let R = $\begin{bmatrix} (36,36,36) & (8,36,51) & (0,0,0) \\ (0,0,0) & (0,0,0) & (18,36,72) \\ (3,36,12) & (0,0,0) & (96,36,66) \\ (0,0,0) & (6,36,6) & (0,0,0) \\ (6,36,6) & (0,0,0) & (0,0,0) \\ (51,36,81) & (0,0,0) & (36,36,36) \\ (0,0,0) & (8,16,51) & (0,0,0) \\ (0,0,0) & (0,0,0) & (69,36,69) \end{bmatrix}$ be the neutrosophic triplet groups matrix from the semigroup S = $\{Z_{105}, \times\}$ associated with the neutral element 36.

Let x = ((9, 36, 39), (0, 0, 0), (39, 36, 9), (36, 36, 36), (0, 0, 0), (0, 0, 0), (27, 36, 48), (0, 0, 0)) and y = ((18, 36, 72), (0, 0, 0), (27, 36, 48)) be neutrosophic triplet groups row matrices.

a) Using x as the initial neutrosophic triplet group row matrix find its effect on the neutrosophic triplet group relational dynamical system R and find the neutrosophic triplet group fixed point pair or a limit cycle pair on x.

b) Find the neutrosophic triplet group fixed point pair or a limit cycle pair using the neutrosophic triplet group row matrix y on R.

c) Transform the resultants pairs in (b) and (c) using T_{ntg}^{-1} into MOD vector pairs.

d) Obtain any other special features associated with this R.

7. Build orthogonal neutrosophic triplet group codes using Z_{3289} of length 8 with 5 number of message symbols.

   a) How many sets of such dual pairs can be got.

8. Find the neutrosophic triplet groups graphs associated with A = {collection of all neutrosophic triplet groups associated with the neutral element 30 of Z_{58}}, the group of neutrosophic triplet groups.

9. Find the graph associated with D_{58}.

10. Characterize all groups whose graph are triangle line graph.

11. Can we have classical groups of neutrosophic triplet groups whose associated graphs are not triangle line graphs?

12. Find the classical groups of the neutrosophic triplet groups associated with Z_{638}.

    a) Which of the classical groups of neutrosophic triplet groups contribute to triangle line graphs.

    b) How many such classical groups of the neutrosophic triplet groups exist?

13. Let S = $\{Z_{247}, \times\}$ be the semigroup under product modulo 247.

    a) Find all the neutrosophic triplet groups associated with each of the neutrals of Z_{243}.

b) Find the respective classical groups of neutrosophic triplet groups and their related graphs.

14. Given $\{Z_n, \times\}$ the semigroup under product modulo n; n = p · q; p and q two distinct primes.

Can the line - triangle graphs of the classical groups of neutrosophic triplet groups have more number of lines than triangles? Justify

15. Given G is a line - triangle graph given by the following Figure 4.16.

G =

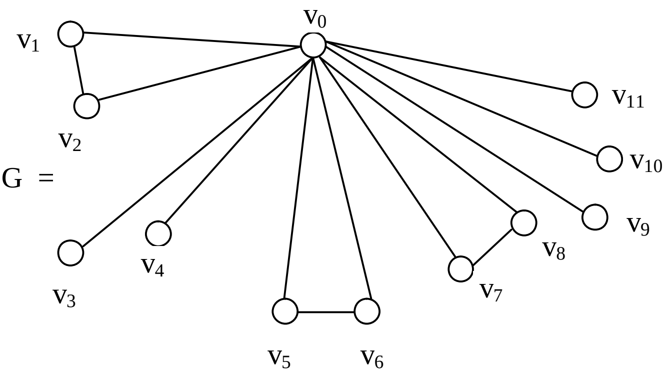

Figure 4.16

Can we have a classical group of neutrosophic triplet groups graph identical with G or isomorphic with G?

16. Can their be a classical group of neutrosophic triplet groups whose graph which is isomorphic with G?

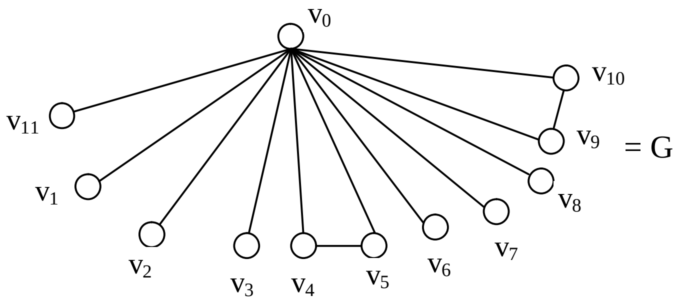

Figure 4.17

17. Can we say all triangle - line graph of order 12 must only be the form given in Figure 4.17 if they are to be associated with the classical group of neutrosophic triplet groups?

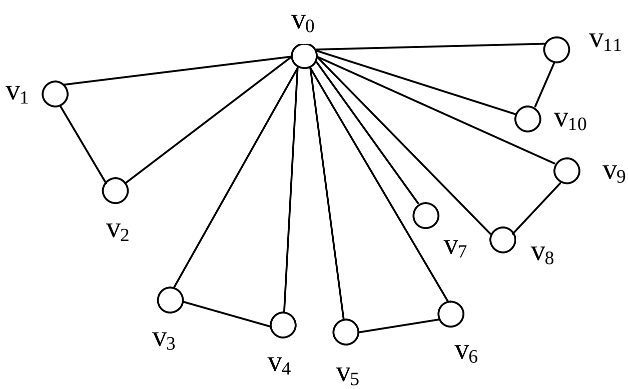

Figure 4.18

18. Characterize all those semigroups $\{Z_n, \times\}$ which can yield at least one new line triangle graph for the graph of the classical group of neutrosophic triplet groups.

19. Can we say there exists only triangle graphs that is no lines associated with the classical group of neutrosophic triplet groups related to $\{Z_n, \times\}$?

20. Let G be the triangle graph given by the following figure.

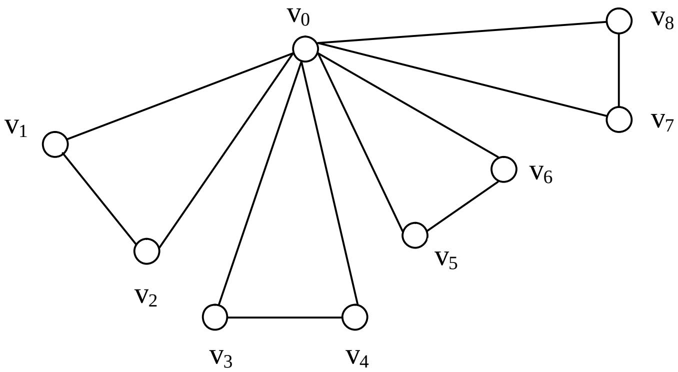

Figure 4.19

Justify your claim!

(Hint use the fact in Z_{pq} the classical group of neutrosophic triplet groups can be of order p – 1 or q – 1 both even we have a group whose graph is of the form given below.

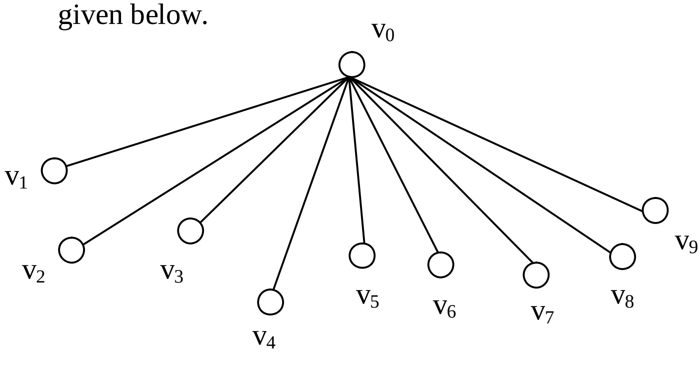

Figure 4.20

21. Let S $=\{Z_{100}, \times\}$ be the semigroup under product modulo 100.

    i) Find all neutral elements of S.

    ii) Can any of the neutral of Z_{100} contribute to classical group of neutrosophic triplet groups which is cyclic?

    iii) How many nontrivial neutrosophic triplet graphs are possible associated with these neutral element of Z_{100}?

    v) Is every neutrosophic triplet group trivial? Justify.

22. Let S = $\{Z_{432}, \times\}$ be the semigroup under product.

    i) Find all the neutrals of Z_{432}.

    ii) Find all those neutrals which contribute to nontrivial neutrosophic triplet groups collection.

    iii) Can any one of the nontrivial neutrosophic triplet groups form a classical cyclic group?

    iv) Does any of the neutrals give only nontrivial quasi neutrosophic triplet pairs?

    vi) Enumerate any of the special features associated with this Z_{432}.

23. Let S $=\{Z_{1280}, \times\}$ be the semigroup under product modulo 1280.

    i) Find all the neutral elements of Z_{1280}.

    ii) Find nontrivial neutrosophic trivial groups associated with neutrals of Z_{1280}.

    iii) Does any of the neutrals contribute to classical group of neutrosophic triplet groups?

    iv) Obtain any other special feature associated with Z_{1280}.

24. Can there be classical groups of neutrosophic triplet groups whose graph are isomorphic with the following graphs for any appropriate n of Z_n.

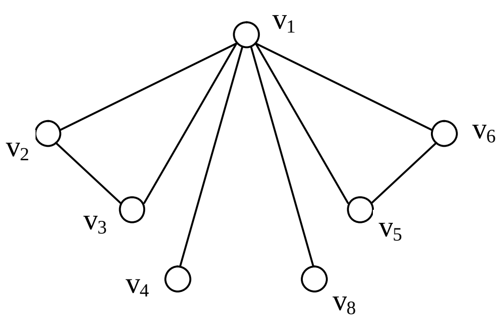

Figure 4.21

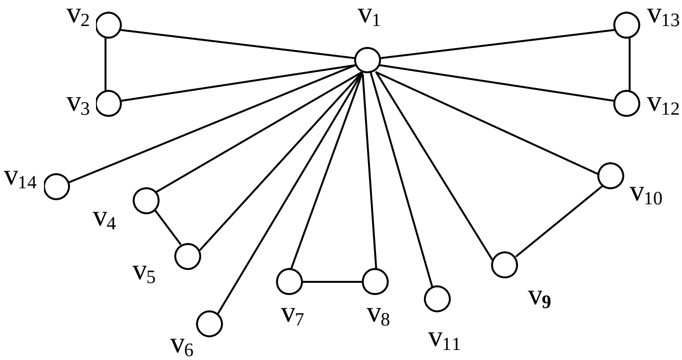

Figure 4.22

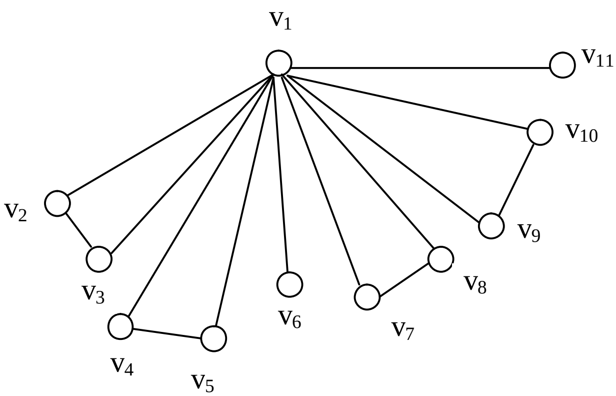

Figure 4.23

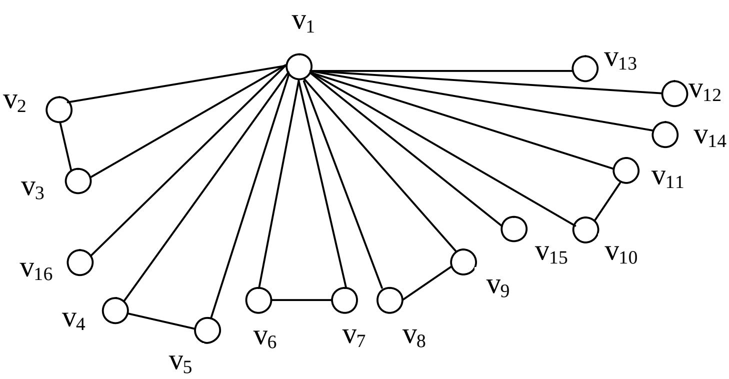

Figure 4.24

25. Prove for the $S = \{Z_{2p}, \times\}$ p and odd prime the classical group of neutrosophic triplet groups G associated with p + 1 becomes a semiring under max and product operation.

    i) Prove G is a semifield.

    ii) Can G be a S-semifield?

    iii) Enumerate all special features associated with G.

26. Let M = {collection of all m × m matrices with entries from $G \cup \{(0, 0, 0)\}$} be the neutrosophic triplet groups of matrices.

    i) Show for all $x \in X = \{(a_1, \ldots, a_m)$ where $a_i \in G \cup \{(0, 0, 0)\}; 1 \leq i \leq m\}$ be the neutrosophic triplet group row matrix, we have c.n max product {x, A}; $A \in M$ converges either to a fixed point or a limit cycle.

    ii) What is the maximum number of iterations to arrive at a fixed point at a resultant?

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Dr.W.B.Vasantha Kandasamy is a Professor in the School of Computer Science and Engg, VIT University, India. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 694 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 122nd book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book authors mainly study the conditions under which the collection of neutrosophic triplet groups associated with a neutral element forms a classical group. Using their properties neutrosophic triplet group codes are defined and the corresponding dual codes occur naturally. The number of dual codes are defined and described.
The graphs of the classical group of neutrosophic triplet groups are given. Finally akin to FCMs model, NCMs model, FRMs model and NRMs model, Neutrosophic Triplet groups Cognitive Maps model and Neutrosophic Triplet groups Relational Maps model are developed and described.
Several open problems are also proposed.

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